Stability and Wave Transmission at Low-Crested Rubble-Mound Structures

By Jentsje W. van der Meer[1] and Ivar F. R. Daemen[2]

Abstract: Low-crested structures can be classified in three categories: dynamically stable reef breakwaters, statically stable low-crested structures with their crest above still-water level (SWL), and statically stable submerged structures. This paper presents practical design formulas and graphs with respect to the stability for each of the three classes. In addition, formulas were developed to predict wave transmission over low-crested rubble mound structures, taking into account the crest height and width, wave height and wave steepness. Most available data sets of various investigations from all over the world were reanalyzed in order to produce the design formulas. The reliability of each formula is described.

INTRODUCTION

In many cases of rubble-mound structure design, a certain degree of overtopping is acceptable, leading to considerable savings on the quantity of material being used. Other structures are so low that under daily wave and water-level conditions the structure is overtopped. Structures with the crest level around still-water level (SWL) and sometimes far below SWL will always allow wave overtopping and transmission.

It is obvious that if the crest level of a structure is low, wave energy can pass over it. This brings about two effects. Firstly, the armor on the front side can be made less heavy compared with a non- or marginally overtopped structure, due to the fact that part of the energy is transmitted over the structure. This means that wave forces during runup and rundown become smaller.

The second effect is that both the crest and rear should be armored in such a way that they can withstand the attack by overtopping waves. For rock structures often the same armor is applied on front face, crest, and rear. The methods to establish the rock armor size for these structures will be given first. These methods, however, do not hold for structures with an armor layer of concrete units. In those cases, it may even be possible that heavier armor units are required on the rear than on the front side. For those structures, physical model investigations may give an acceptable solution. The other design concern, wave transmission, will also be dealt with in this paper.

The complete reanalysis of the data on stability can be found in Van der Meer (1990a). A summary was presented by Van der Meer and Pilarczyk (1990). The data sets on wave transmission are described in Van der Meer (1990b), the analysis leading to practical formulas in Daemen (1991). A

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summary on transmission was presented by Van der Meer and d'Angremond (1991). This paper is a summary of the important results from these sources.

CLASSIFICATION OF LOW-CRESTED STRUCTURES

Low-crested rock structures can be divided into three categories: dynamically stable reef breakwaters, statically stable low-crested structures (with the crest above the still-water level, and statically stable submerged breakwaters.

A reef breakwater is a low-crested homogeneous pile of stones without a filter layer or core which is allowed to be reshaped by wave attack (Fig. 1). The initial crest height is just above the water level. Under severe wave conditions, the crest height reshapes to a certain equilibrium crest height. The equilibrium crest height and corresponding wave transmission are the main design parameters.

Statically stable low-crested breakwaters are close to non- or marginally overtopped structures, but are more stable due to the fact that a (large) part of the wave energy can pass over the breakwater (Fig. 2).

All waves overtop statically stable submerged breakwaters, and the stability increases remarkably as the crest height decreases (Fig. 3). It is obvious that the wave transmission is substantial at these structures.

DESCRIPTION OF DATA SETS

A number of studies on stability and wave transmission has been published with enough details to make a comparison with other studies useful and possible. All data selected, except for one, were obtained with random-wave testing. Data and references based on monochromatic waves were not taken into account as they were found to be too far from reality. A short description of the data sets will be given in the following section.

A very extensive investigation on stability of rock slopes and gravel beaches was performed at Delft Hydraulics between 1983 and 1987. The (basic) background and all test data were described in Van der Meer (1988). A

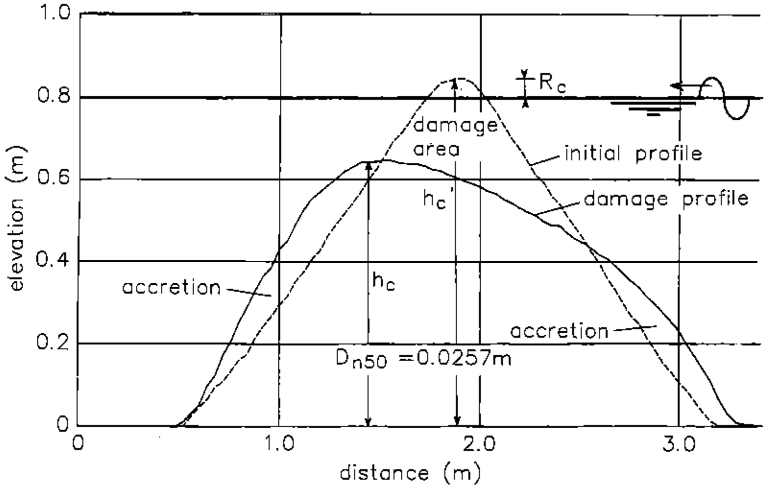

FIG. 1. Example of Reef-Type Breakwater

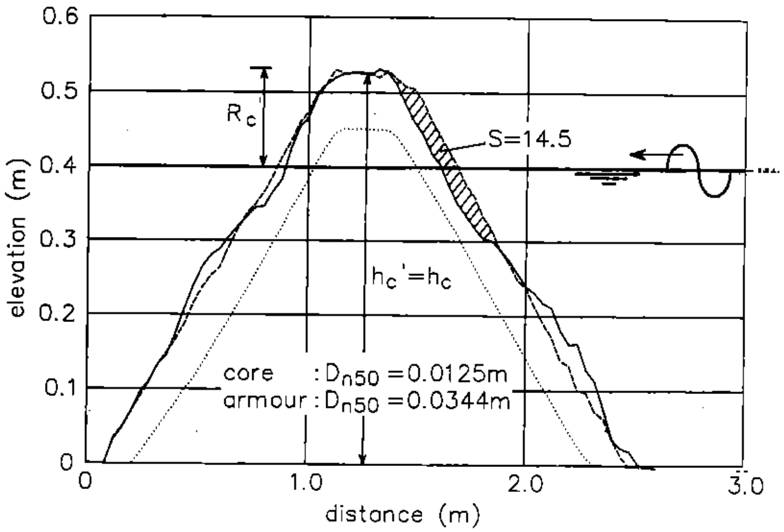

FIG. 2. Example of Low-Crested Breakwater

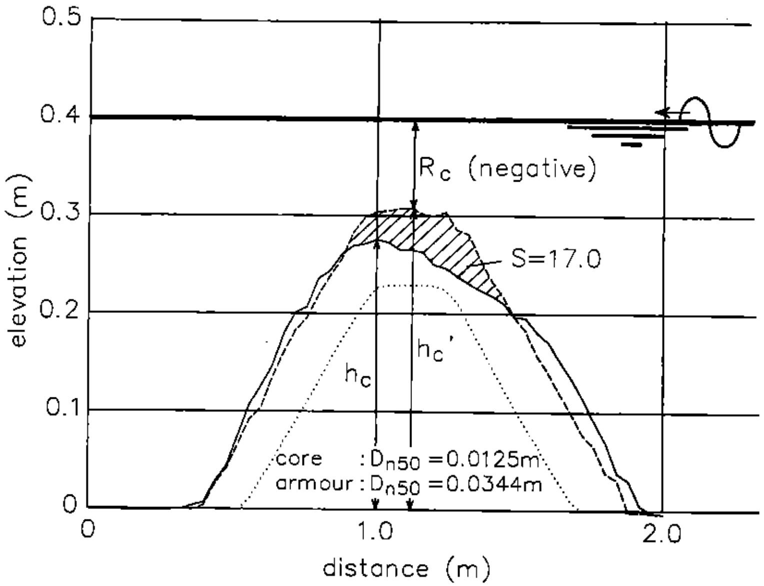

FIG. 3. Example of Submerged Breakwater

part of the study aimed at stability and transmission at low-crested breakwaters. These tests cover all three structure types previously described (reef type, low-crested above SWL, and submerged).

Ahrens (1987) investigated the structure stability and wave transmission for reef-type breakwaters (see Fig. 1). During his tests on wave transmission it may well be possible that the crest height had changed, which makes it difficult to choose the correct crest height for that test. The crest height from Ahrens' tests used in this paper is the height measured after the test. Ahrens performed a large number of tests on stability and transmission at these structures and presented a formula for the equilibrium crest height. Hearn (1987) gives a more extensive analysis of Ahrens' data, and she developed a design formula for wave transmission.

Powell and Allsop (1985) describe the hydraulic performance of low-crested breakwaters with the crest above SWL, including transmission. Only a small amount of damage, namely displacement of some rocks, was allowed during design conditions.

Givler and Sørensen (1986) described about 45 tests on the stability of submerged breakwaters. The tests were performed with periodic waves and included both a large range of wave heights and wave periods. This was the reason to select this case, as other data sets with random waves were not available. The damage at the crest was measured and the damage criteria for design are similar to conventional breakwaters (no or only little damage allowed). Only the results on stability are reanalyzed in this paper.

Seelig (1980) has measured wave transmission for a large number of structure cross sections, mostly with periodic waves, but also with random waves. For the reason just described, only the random wave data have been considered. This data set was only used for wave transmission.

Three types of structures were tested by Daemrich and Kahle (1985), all with the crest at or below the water level. Only wave transmission was observed during the tests.

Finally, Daemen (1991) described tests on wave transmission at statically stable structures with the crest around SWL. Tests were performed with constant wave steepness and various crest levels and wave heights.

STABILITY OF LOW-CRESTED STRUCTURES

Reef Breakwaters

The stability analysis conducted by Ahrens (1987, 1989) and Van der Meer (1990a) were concentrated on the change in crest height due to wave attack (see Fig. 1). Ahrens defined a number of dimensionless parameters that described the behavior of the structure. The main parameter being the relative crest height reduction factor h_c/h_c', which is the ratio of the crest height at completion of a test, h_c, to the height at the beginning of the test, h_c'. The natural limiting value of h_c/h_c' are 1.0 (no deformation) and 0.0 (structure not present anymore), respectively.

The wave height can be characterized by $H_s/\Delta D_{n50}$ (Van der Meer 1988) or N_s [stability number from Ahrens (1987, 1989)]

$$\frac{H_s}{\Delta D_{n50}} = N_s \qquad (1)$$

where H_s = significant wave height, H_s or H_{m0} ($H_{m0} = \sqrt{4m_0}$ was used in this study); Δ = relative mass density; $\Delta = \rho_a/\rho_w - 1$; ρ_a = mass density of armour rock; ρ_w = mass density of water; D_{n50} = nominal diameter of rock [$D_{n50} = (M_{50}/\rho_a)^{1/3}$]; M_{50} = average mass (50% value on mass distribution curve); and m_0 = zeroth moment of wave energy density spectrum.

For the reef breakwater, Ahrens found that a longer wave period caused more displacement of the material than a shorter period. Therefore, he introduced the spectral (or modified) stability number, N_s^*, defined by:

$$N_s^* = \frac{H_s^{2/3}L_p^{1/3}}{\Delta D_{n50}} \qquad (2)$$

where L_p = Airy wave length calculated using the peak period of the wave-energy density spectrum (T_p) and the water depth at the toe of the structure (h). In fact, a local wave steepness is introduced in (2), and the relationship between the stability number N_s and the spectral stability number N_s^* can simply be given by

$$N_s^* = \frac{N_s}{s_p^{1/3}} = \frac{H_s}{\Delta D_{n50}}\frac{1}{s_p^{1/3}} \qquad (3)$$

where s_p = local wave steepness; $s_p = H_s/L_p$.

That a longer wave period should give more damage than a shorter period is not always true. Ahrens concluded that it was true for reef breakwaters where the crest height lowered substantially during the test. It is, however, not true for non- or marginally overtopped breakwaters (Van der Meer 1987, 1988). The influence of the wave period in that case is much more complex than suggested by (3).

The crest height (reduction) of a reef-type breakwater can be described by

$$h_c = \sqrt{\frac{A_t}{\exp(aN_s^*)}} \qquad (4)$$

where a = coefficient; and A_t = area of structure cross section.

Ahrens presented various equations for the coefficient a. The most recent and refined one is given by Ahrens (1989)

$$a = 0.046\left(\frac{h_c' - h}{h}\right) + 0.2083\left(\frac{h_c}{h}\right)^{1.5} - 0.144\left(\frac{h_c}{h}\right)^2 + \frac{0.4317}{\sqrt{B_n}} \qquad (5a)$$

where h = water depth at structure toe; and

$$B_n = \frac{A_t}{D_{n50}^2}\ \text{(bulk number)} \qquad (5b)$$

The structures of Van der Meer (1988) had other crest heights, water depths, bulk numbers, and slope angles than Ahrens' structures. A fit of (4) and (5) with these data showed that they could not describe these additional data. The difference was large, and results were presented in Van der Meer (1990a). Eqs. (4) and (5), therefore, can only be used for reef-type breakwaters that are similar to Ahrens' cross sections.

Therefore, all the data of Ahrens (1987) were reanalyzed together with the data of Van der Meer (1988). The complete analysis is given by Van der Meer (1990a). The analysis showed that the breakwater response slopes C' (as initially built) and C (after the test) had to be included. The breakwater response slopes are defined by

$$C' = \frac{A_t}{h_c'^2} \qquad (6a)$$

and

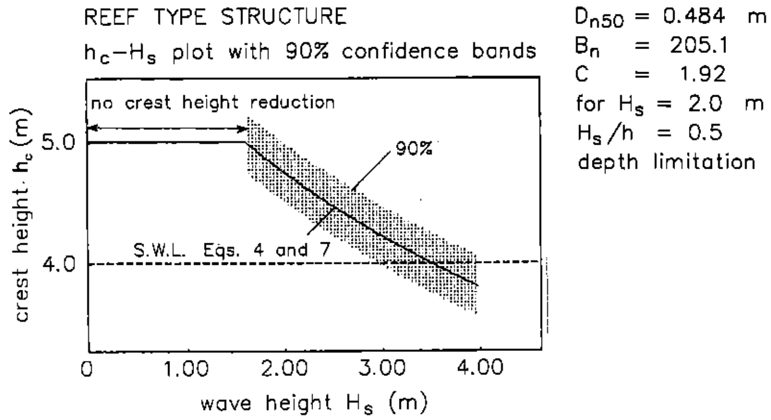

FIG. 4. Design Graph of Reef-Type Breakwater

$$C = \frac{A_t}{h_c^2} \quad (6b)$$

The final equation that was derived from the analysis is given by (4) with

$$a = -0.028 + 0.045C' + 0.034(h_c'/h) - 6.10^{-9}B_n^2 \quad (7)$$

and $h_c = h_c'$ if h_c in (4) is greater than h_c'.

The lowering of the crest height of reef-type structures, as shown in Fig. 1, can be calculated with (4) and (7). It is possible to draw design curves from these equations that give the crest height as a function of N_s^* or even H_s. An example of h_c versus H_s is shown in Fig. 4. The reliability of (4) can be described by giving the 90% confidence bands given by $h_c \pm 5\%$, and is shown in Fig. 4. In Fig. 4, M_{50} = 300 kg; ρ_a = 2,650 kg/m^3; ρ_w = 1,025 kg/m^3, h_c' = 5.0 m, h = 4.0 m; A_t = 48.0 m^2; and T_p = 8.00 s.

Statically Stable Low-Crested Breakwaters above SWL

The stability of a low-crested conventional breakwater can be related to the stability of a non- or marginally overtopped structure. Stability formulas, such as the Hudson formula or more advanced formulas (Van der Meer 1987, 1988) can be used. The required rock armor diameter for an overtopped breakwater can then be determined by application of a reduction factor for the mass of the armor.

Data sets that could be used for analysis were a part of Ahrens' data (with small damage to the crest), Powell and Allsop (1985), and Van der Meer (1988). Fig. 5 gives the damage curves of a part of Van der Meer's tests with four crest heights, R_c, for a constant wave period of 1.7 s. From this figure, it is obvious that a decrease in structure crest height results in an increase in stability, although the difference between no overtopping and little overtopping (R_c = 0.125 m) is small. In Fig. 5, cot α = 2; N = 3,000; D_{n50} = 0.0344 m; and T_m = 1.7 s.

Furthermore, from the tests it could be concluded that the wave period had an influence on the maximum relative crest level R_c/H_s. For higher

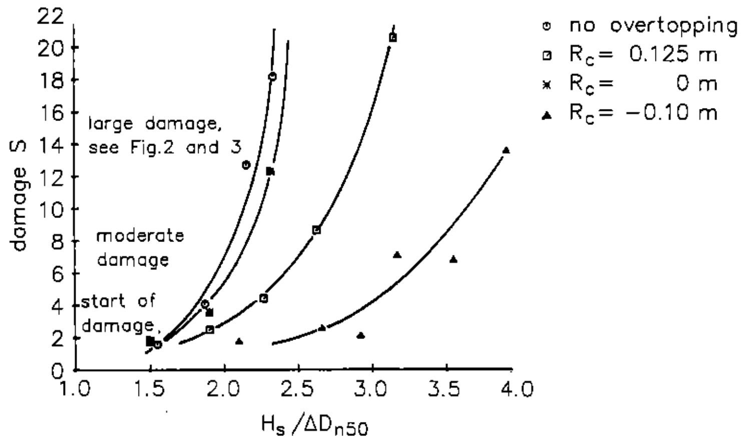

FIG. 5. Influence of Crest Height on Damage Curves (Van der Meer 1988)

values of R_c/H_s, the structure behaved as a nonovertopping one. This can also be explained in a physical way. A long period gives a higher runup on a slope than a short period. Therefore, more energy is lost by overtopping for a long period at the same crest level as for a short period.

The transition crest height where the increase in stability begins (given as a R_c/H_s value) should in fact also be a function of the wave period (or wave steepness). In Powell and Allsop (1985), a dimensionless crest height R_p^* was introduced that was used to describe wave transmission and that included the wave steepness. The definition is given by

$$R_p^* = \frac{R_c}{H_s}\sqrt{\frac{s_{op}}{2\pi}} \quad (8)$$

where s_{op} = fictitious wave steepness = $2\pi H_{m0}/gT_p^2$.

Curve fitting showed that the transition crest height, where for lower values the stability increases, can simply be described by

$$R_p^* = 0.052 \quad (9a)$$

or

$$\frac{R_c}{H_s} = \frac{0.13}{\sqrt{s_{op}}} \quad (9b)$$

The average increase in stability ($H_s/\Delta D_{n50}$ or N_s^*) for a structure with the crest at SWL, in comparison with a nonovertopped structure, is of the order of 20–30%. If the increase in stability is set at 25%, independent of wave steepness, and when a linear increase in stability is assumed between R_p^* = 0.052 and R_p^* = 0, the increase in stability can be described as a function of R_p^* only (or R_c/H_s and s_{op}), see Fig. 6.

In addition, the reduction in required nominal diameter D_{n50}, becomes

$$\text{Reduction factor for } D_{n50} = \frac{1}{1.25 - 4.8R_p^*} \quad (10)$$

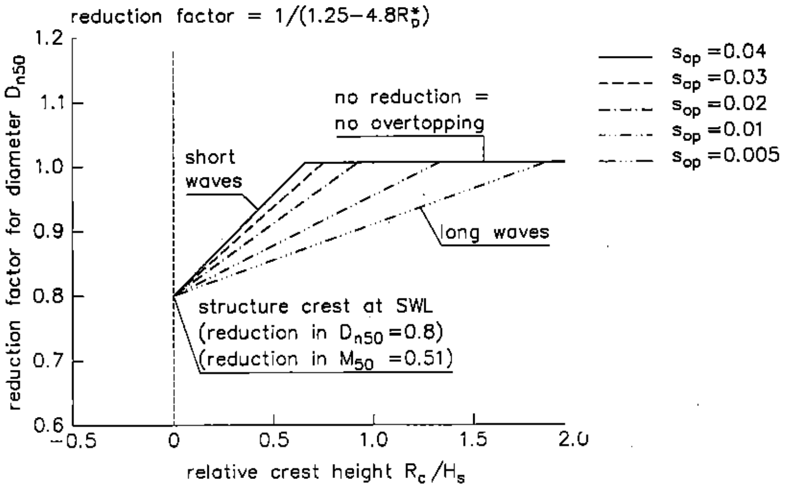

FIG. 6. Design Graph with Reduction Factor for Rock Diameter of Low-Crested Structure ($R_c < 0$) as Function of Relative Crest Height and Wave Steepness

for $0 < R_p^* < 0.052$.

This final (10) describes the stability of a statically stable low-crested breakwater with the crest above SWL simply by application of a reduction factor on the required diameter of a nonovertopped structure [Van der Meer (1987)]. Eq. (10) is shown in Fig. 6 for various wave steepnesses, and can be used as a design graph. The reduction factor to be applied for the nominal diameter can be read from this graph [or calculated by using (10)].

An average reduction of 0.8 in diameter is obtained for a structure with the crest height at the water level. The required mass in that case is a factor $0.8^3 = 0.51$ of that required for a nonovertopped structure.

It is not really required to describe the reliability of the reduction factor in (10). The reliability of D_{n50} is about the same as for a nonovertopped structure, i.e., the reliability depends on the stability formula that is used to calculate the D_{n50} for a nonovertopped structure. These reliabilities are described in Van der Meer (1988).

SUBMERGED BREAKWATERS

During their tests, Ahrens (1987, 1989), Allsop (1983), and Powell and Allsop (1985) always had an initial crest level at or above SWL. Only Van der Meer (1988) and Givler and Sørensen (1986) had initial crest heights below SWL. The total amount of data is limited, however. Van der Meer (1988) tested only a slope angle of 1:2 and Givler and Sørensen (1986) tested only a slope of 1:1.5. The seaward slope angle may have some influence on the stability of the submerged structure. Therefore, the description of submerged structures here will be only valid for this range of slopes, say about 1:1.5 – 1:2.5.

The slope angle has large influence on nonovertopped structures. In the case of submerged structures, the wave attack is concentrated on the crest and less on the seaward slope. Therefore, excluding the slope angle of submerged structures (as it is a governing parameter for stability) may be legitimate.

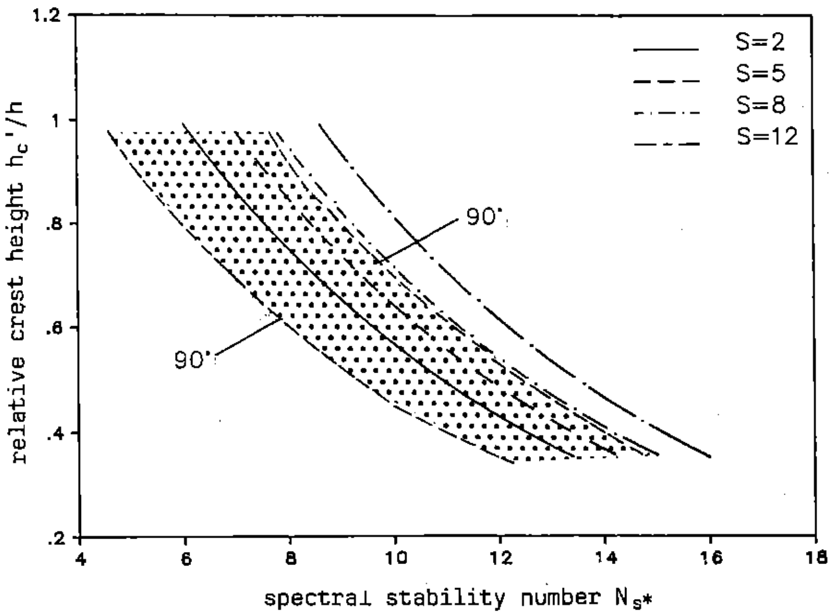

FIG. 7. Design Curves for Submerged Structure with 90% Confidence Bands for $S = 2$

The stability of submerged breakwaters appeared only to be a function of the relative crest height h_c'/h, the damage level S, and the spectral stability number N_s^*. The damage level S is defined by Van der Meer (1988). Briefly, $S = 2$ means start of damage, $S = 5$ is moderate damage, and $S = 8$–12 means severe damage (filter layer visible; not acceptable). Figs. 2 and 3 give examples of large S-values of 14.5 and 17.0. The final design formula is given by

$$\frac{h_c'}{h} = (2.1 + 0.1S)\exp(-0.14N_s^*) \quad (11)$$

For fixed crest height, water level, damage level, and wave height and period, the required ΔD_{n50} can be calculated from (11), finally yielding the required rock weight. Also, wave height versus damage curves can be derived from (11). Eq. (11) is shown as a design graph in Fig. 7 for four damage levels. The reliability of (11) can be described when the factor 2.1 is considered as a stochastic variable. The data gave a standard deviation of 0.35. With this standard deviation, it is possible to calculate the 90% confidence bands, using $2.1 \pm 1.64 \times 0.35$ in (11). Fig. 7 gives the 90% confidence bands for $S = 2$. The scatter is quite large, and this should be considered during the design of submerged structures.

WAVE TRANSMISSION AT LOW-CRESTED STRUCTURES

Governing Variables

The most important variables with respect to wave transmission are summarized here and explained in Fig. 8.

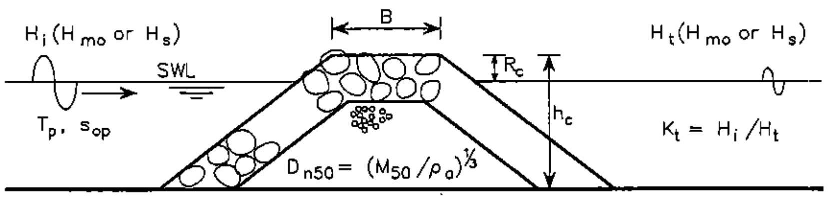

FIG. 8. Governing Variables Related to Wave Transmission

The crest height related to SWL, i.e., the crest freeboard is given by R_c. In particular, if the size of the armor rock is large and the crest level is close to SWL (i.e. R_c close to zero), the definition of the crest level is crucial. From the existing data sets, it could not be verified in what way the crest level was defined. For the test series of Daemen (1991), the crest has been defined as the plane through the upper edges of the armor units. The height of the structure is defined by h_c. The crest width is defined by B. The water depth in front of the structure is given by h. The relationship between the parameters is $R_c = h_c - h$.

The size of the armor rock is introduced as the nominal diameter D_{n50}. The wave heights are given by H_i and H_t, representing the incoming and transmitted wave height, respectively. Both values are expressed as H_s (mean of highest one-third of the waves), or H_{m0} (based on spectrum, $\sqrt{4m_0}$). It must be emphasized here that H_t is not always Rayleigh distributed. The wave period, T_p, is used throughout this part on wave transmission, being the peak period of the spectrum; the wave steepness s_{op} is defined as $2\pi H_s/gT_p^2$, with H_s defined at the toe of the structure. The transmission coefficient K_t is given by $K_t = H_t/H_i$. Finally, when discussing transmission of wave energy, it may be necessary to account for the permeability of the structure.

Important references on wave transmission are Seelig (1980) and Madsen and White (1976). Seelig (1980) describes a model for wave transmission, supported by many tests. Most of the tests were performed with monochromatic waves and the model was based only on these tests. Random wave tests showed fairly good agreement when the mean wave height and the peak period were used, but the model was not based on these random wave tests. Seelig (1980) used the results of Madsen and White (1976) to describe the transmission through a breakwater. These results were only based on monochromatic waves and basically for (very) long waves. Comparison with short waves gave large overpredictions in transmission coefficients ($K_t = 0.16$ versus 0.04).

The model of Seelig (1980) may be useful for design. In this paper it was not used, as it was too much based on monochromatic and long waves. However, the tests with random waves of Seelig (1980) were part of the total data set that was reanalyzed.

Analysis with R_c/H_i as Main Parameter

Van der Meer (1990b) attempted to analyze the existing data starting from the assumption that the transmission coefficient K_t would largely depend on a dimensionless crest height. The dimensionless crest height was defined in two ways: as R_c/H_i; namely, only related to the wave height; and as the parameter R_p^* [(8)], which includes both wave height and steepness, used by Powell and Allsop (1985). However, they used the mean wave

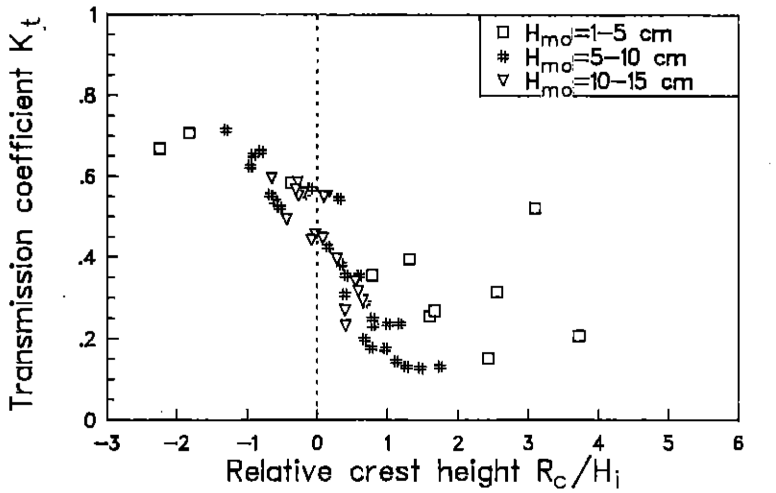

FIG. 9. Influence of Wave Height on Transmission; T_p = 1.45 s (Ahrens 1987)

period T_m, instead of T_p, in calculating the wave steepness. It is stressed again that in this part of wave transmission only the peak period is used.

From the analysis with R_c/H_i and R_p^* versus K_t, it could in general be concluded that the parameter R_p^* is not better than R_c/H_i, as long as the whole range of relative crest levels is considered. Only for positive values of R_p^* ($\geq$ 0.025) the results are better than with R_c/H_s.

Another phenomenon was found from Ahrens' (1987) data. Plotting all data of Ahrens obtained for one particular wave period against the relative crest height, a wide scatter of K_t was observed for high values of R_c/H_i (see Fig. 9). A closer analysis shows that this scatter is mainly due to the occurrence of low wave heights, having roughly the same dimensions as the rock. Apparently low (and relatively long) waves travel easily through the top of the structure.

Combining all data, and plotting the transmission coefficient against the relative crest height, Fig. 10 is obtained. As expected, the result shows considerable scatter, but a clear trend can be observed. Part of the scatter can be attributed to the influence of the wave period, and part to the influence of extremely small waves; crest width and permeability may have some influence as well.

The average value of K_t for $-2 < R_c/H_i < -1$ is about 0.8. Except for the triangles (Ahrens' data, small wave heights), the average value of K_t for $1 < R_c/H_i < 2$ is about 0.1. Between these ranges, the value of K_t decreases almost linearly with R_c/H_i. Based on this simple analysis, the following formula for wave transmission can be proposed:

$$\text{For } -2.0 < \frac{R_c}{H_i} < -1.13 \qquad K_t = 0.80 \qquad (12a)$$

$$\text{For } -1.13 < \frac{R_c}{H_i} < 1.2 \qquad K_t = 0.46 - 0.3\frac{R_c}{H_i} \qquad (12b)$$

$$\text{For } 1.2 < \frac{R_c}{H_i} < 2.0 \qquad K_t = 0.10 \qquad (12c)$$

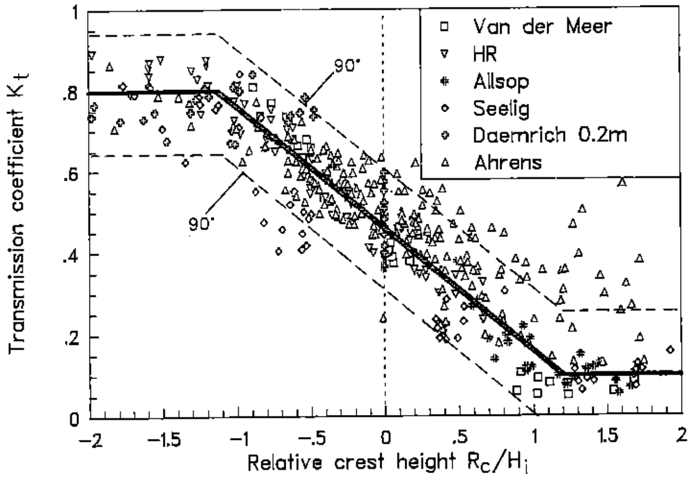

FIG. 10. Wave Transmission versus Relative Crest Height (All Data with 90% Confidence Bands)

This curve is shown in Fig. 10 as well. The scatter is large, which means that the formula can be simple. It means also that for application this large scatter should be taken into account. The standard deviation of K_t amounted to $\sigma(K_t) = 0.09$ and was assessed from the graph, assuming a normal distribution. This means that the 90% confidence levels are given by $K_t \pm 0.15$. It is evident that for large negative values of R_c/H_i, K_t should approach one, and for large positive values of R_c/H_i, a value close to zero. The 90% confidence levels are given in Fig. 10.

For design purposes, however, the scatter is a serious drawback. After all, an accurate forecast of wave transmission may lead to considerable savings by reducing the total height of the structure. Therefore, it was decided to perform additional tests and to continue the efforts toward a better expression for wave transmission.

Analysis with R_c, s_{op} and D_{n50} as Main Parameter

To study the wave-transmission process further, additional tests were carried out by Daemen (1991). The results of the tests are presented in Fig. 11, in which a distinction has been made between the two wave steepness values of $s_{op} = 0.02$ (long waves) and 0.04 (short waves).

Until now, wave transmission has been described in the conventional way as a function of R_c/H_i. It is not clear, however, that the use of this combination of crest freeboard and wave height produces similar results with, on the one hand constant R_c and variable H_i, and on the other hand variable R_c and constant H_i. Moreover, when R_c becomes zero, all influence of the wave height is lost, leading to a large scatter in the graph at $R_c = 0$. Therefore, it was decided to separate R_c and H_i.

There is a direct relationship between the design wave height and the size of armor rock, which is often given as the stability factor $H_s/\Delta D_{n50}$. It can be concluded that the nominal diameter of the armor rock will characterize the rubble-mound structure. It is, therefore, also a good parameter to characterize both the wave height and the crest height in a dimensionless way.

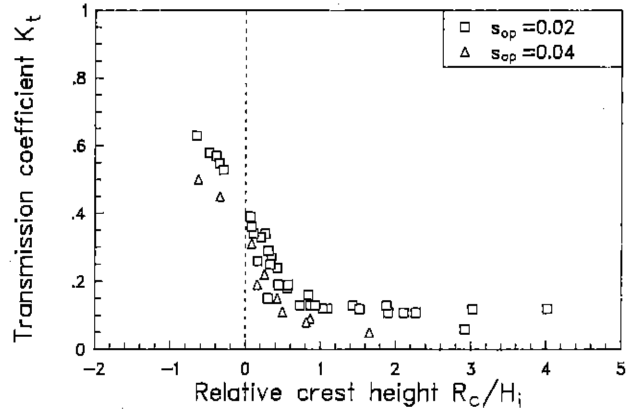

FIG. 11. Wave Transmission Data of Daemen (1991)

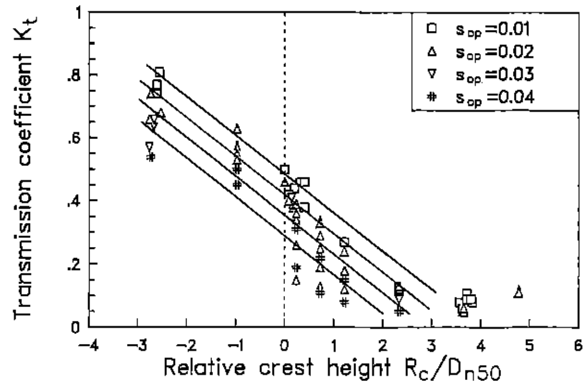

FIG. 12. Wave Transmission versus R_c/D_{n50} (Van der Meer 1990b; Daemen 1991)

The relative wave height can then be given as H_i/D_{n50}, in accordance with the stability factor, and the relative crest height as R_c/D_{n50}, as the number of rocks that the crest level is above or below SWL. Moreover, a separation into H_i/D_{n50} and R_c/D_{n50} enables a distinction between various cases. For example, low H_i/D_{n50} values (smaller than 1–2) produce low waves traveling through the crest, and high H_i/D_{n50} values (3–5) yield situations under extreme wave attack. Finally, D_{n50} can be used to describe other breakwater properties, such as the crest width B. This yields the parameter B/D_{n50}.

The primary parameters for wave transmission can now be given as: Relative crest height R_c/D_{n50}; relative wave height H_i/D_{n50}; fictitious wave steepness s_{op}; and possibly B/D_{n50}. Fig. 12 shows the wave transmission versus R_c/D_{n50} for the data of Van der Meer (1990b) and the tests of Daemen (1991). The data are grouped by constant wave steepness s_{op}. Straight lines are drawn through the points with the same wave steepness. Fig. 12 makes clearly visible that a lower wave steepness (or a longer period) results in a larger transmission coefficient. This is true for the whole area of R_c/D_{n50}, except for large positive and negative values. Furthermore, the lines in Fig. 12 are parallel to each other.

The general trend of the wave-transmission coefficient, going from high positive values of R_c/D_{n50} to high negative values, is that the transmission coefficient first remains low, then increases in the area of R_c/D_{n50} between +2 and −2, and finally remains high. Theoretically, the increasing wave-transmission coefficient will be expressed by a smooth curve from zero for very high crest heights to one for very low crest heights. The most important area, however, is the area where the transmission increases rapidly. For the sake of simplicity, it is assumed that this area can be described by straight lines, as shown in Fig. 12.

This means that a linear relationship is assumed in the area of R_c/D_{n50} roughly between +2 and −2. The wave transmission can now be described as

$$K_t = a\frac{R_c}{D_{n50}} + b \quad \text{(13)}$$

In this equation, a determines the slope of the line, and b gives the value of K_t at $R_c/D_{n50} = 0$. From Fig. 12 it can already be concluded that the wave steepness s_{op} is only present in the coefficient b and not in the coefficient a (the lines in Fig. 12 are parallel).

Fig. 13 shows the data of Daemen (1991) with a constant wave steepness of $s_{op} = 0.02$ and various classes of relative wave heights. In fact, Fig. 13 shows the influence of the relative wave height H_i/D_{n50} on wave transmission. From this Fig. 13, it can be concluded that for $R_c/D_{n50} < -1$, a larger H_i/D_{n50} produces smaller wave transmission. For $R_c/D_{n50} > -1$, the opposite occurs: a larger H_i/D_{n50} gives larger wave transmission.

This phenomenon can be explained in a physical way. On a low-crested breakwater, where R_c/D_{n50} is positive, the transmission is primarily determined by overtopping and thus by wave runup. In this area of R_c/D_{n50}, a larger relative wave height yields a higher runup, thus more overtopping, and hence a larger transmission coefficient. On a submerged breakwater, where R_c/D_{n50} is negative, higher waves will be more affected by the structure whereas small waves pass unhindered. In this case, a larger relative wave height results in a smaller transmission coefficient.

Fig. 14 gives the values of the transmission coefficient that holds for high (above SWL) and low (submerged) relative crest heights, outside the range

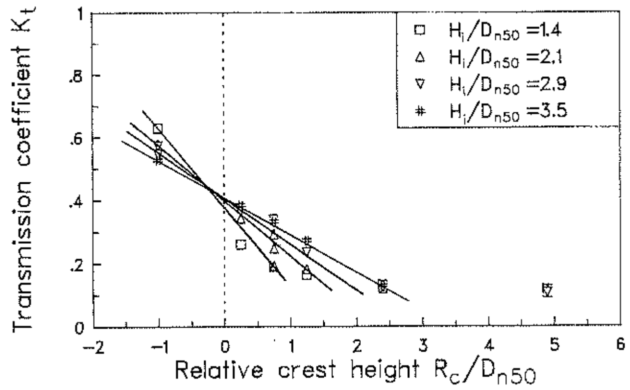

FIG. 13. Influence of Relative Wave Height on Wave Transmission for Constant Wave Steepness of s_{op} = 0.02

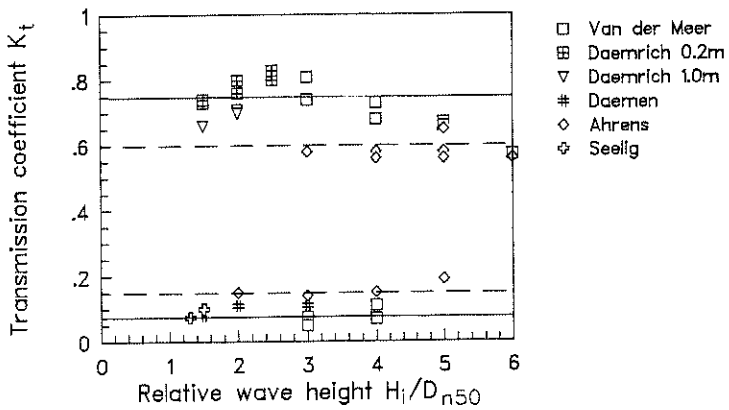

FIG. 14. Minimum and Maximum Wave Transmission

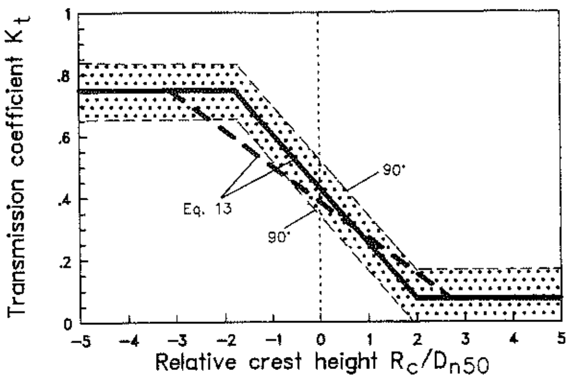

FIG. 15. Basic Graph for Wave Transmission with 90% Confidence Bands

given by the curve of (13). Fig. 14 shows that both the maximum and minimum transmission are independent of the relative wave height H_i/D_{n50}. Based on Fig. 14, the following minimum and maximum values were derived:

Conventional breakwaters

Minimum: $K_t = 0.075$; maximum: $K_t = 0.75$ .... (14)

Reef-type breakwaters

Minimum: $K_t = 0.15$; maximum: $K_t = 0.60$ for $\frac{R_c}{D_{n50}} < -2$

linearly increasing to $K_t = 0.80$ for $\frac{R_c}{D_{n50}} = -6$ .... (15)

Final Results on Wave Transmission

The final outcome of the analysis on wave transmission, including the data of Daemen (1991), was a linear relationship between the wave-transmission coefficient K_t and the relative crest height R_c/D_{n50}, which is valid between minimum and maximum values of K_t. In Fig. 15, the basic graph is shown. The linearly increasing curves are presented by (13) with

$$a = 0.031\, H_i/D_{n50} - 0.24. \quad (16)$$

Eq. (14) is applicable for conventional and reef-type breakwaters. The coefficient b for conventional breakwaters is described by

$$b = -5.42 s_{op} + 0.0323 \frac{H_i}{D_{n50}} - 0.0017 \left(\frac{B}{D_{n50}}\right)^{1.84} + 0.51 \quad (17)$$

and for reef-type breakwaters by

$$b = -2.6 s_{op} - 0.05 \frac{H_i}{D_{n50}} + 0.85 \quad (18)$$

Permeability of the structure (underneath the armor layer) did not show significant influence. In the cases described in this paper, most wave transmission is caused by overtopping or by waves traveling through the armor layer on the crest. The minimum and maximum values are described by (14) and (15).

Validation and Reliability of Formula on Wave Transmission

The analysis was based on various groups with constant wave steepness and a constant relative wave height. The validity of the wave-transmission formula [(13)] corresponds, of course, with the ranges of these groups. The formula is valid for

$$1 < \frac{H_i}{D_{n50}} < 6 \quad (19a)$$

and

$$0.01 < s_{op} < 0.05 \quad (19b)$$

Both upper boundaries can be regarded as physically bound. Values of $H_i/D_{n50} > 6$ will cause instability of the structure, and values of $s_{op} > 0.05$ will cause waves breaking because of steepness. In fact, boundaries are only given for wave heights that are too low relative to the rock diameter and for very low wave steepnesses (low swell waves).

The formula is applicable outside the range just given, but its reliability is lower. Fig. 16 shows the measured wave-transmission coefficient versus the one calculated from (13) for various data sets of conventional breakwaters. The reliability of the formula can be described by assuming a normal distribution around the line in Fig. 16. With the restriction of the range of application previously given, the standard deviation amounted to $\sigma(K_t) = 0.05$, which means that the 90% confidence levels can be given by $K_t \pm 0.08$. This is a remarkable increase in reliability compared with the simple formula given by (12) and Fig. 10, in which a standard deviation of $\sigma(K_t) = 0.09$ was given. The 90% confidence levels are also given in Fig. 15.

The reliability of the formula for reef-type breakwaters is more difficult to describe. If only tests are taken where the crest height had been lowered less than 10% of the initial height h_c', and the test conditions lie within the range of application, the standard deviation amounts to $\sigma(K_t) = 0.031$. If

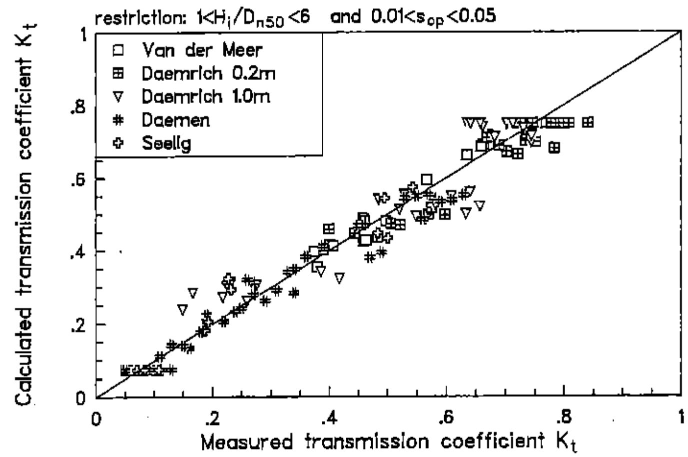

FIG. 16. Calculated versus Measured Wave Transmission for Conventional Breakwaters

the restriction about the crest height is not taken into account, the standard deviation amounts to $\sigma(K_t) = 0.054$.

CONCLUSIONS

Low-crested rubble mound structures can be divided into three categories: dynamically stable reef breakwaters; statically stable low-crested breakwaters ($R_c/H_s > 0$); and statically stable submerged breakwaters. Waves overtop these structures, and the stability increases remarkably if the crest height decreases.

The stability of reef breakwaters is described by (4) and (7). Design curves can be drawn with the aid of these equations. An example is given in Fig. 4.

The stability of a low-crested breakwater with the crest above SWL is first established as a nonovertopped structure. Stability formulas derived by Van der Meer (1987, 1988) can be used. The required rock diameter for an overtopped breakwater can then be determined by application of a reduction factor, given by (10). Design curves are shown in Fig. 6.

The stability of submerged breakwaters depends on the relative crest height, the damage level, and the spectral stability number. The stability is described by (11), and a design graph is given in Fig. 7.

A formula was described for wave transmission at low-crested structures. The outcome of this formula was a linear relationship between the wave-transmission coefficient K_t and the relative crest height R_c/D_{n50}, which is valid between minimum and maximum values of K_t. In Fig. 15 the basic graph is shown. The linearly increasing curves are presented by (13), (16), and (17) (conventional breakwaters) or (18) (reef-type). The minimum and maximum values of K_t are given by (14) and (15).

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A_t = area of structure cross section;
a, b = coefficients;
B = width of structure crest;
B_n = bulk number, $B_n = A_t/D_{n50}^2$;
C, C' = breakwater response slope, after and before a test, [(6)];
D_{n50} = nominal diameter, $D_{n50} = (M_{50}/\rho_a)^{1/3}$;
g = gravitational acceleration;
H_i = incident wave height, H_{m0} of H_s;
H_{m0} = significant wave height, $4\sqrt{m_0}$;
H_s = significant wave height, mean of highest one third of waves;
H_t = transmitted wave height, H_{m0} or H_s;
h = water depth at toe of structure;
h_c, h_c' = structure height, after and before a test;
K_t = transmission coefficient, $K_t = H_t/H_i$;
L_p = local wave length;
M_{50} = 50% value on mass distribution curve;
m_0 = zeroth moment of wave energy density spectrum;
N_s = stability number, $N_s = H_s/\Delta D_{n50}$;
N_s^* = spectral stability number;
P = notional permeability factor;
R_c = crest height above SWL;
R_p^* = dimensionless crest height;
S = damage level;
s_{op} = wave steepness, $s_{op} = 2\pi H_s/gT_p^2$;
s_p = local wave steepness;
T_p = peak wave period;
α = slope angle;
Δ = buoyant mass density, $\Delta = \rho_a/\rho_w - 1$;
ρ_a, ρ_w = mass density rock, water; and
σ = standard deviation of normal distribution.