Performance Design of Maritime Structures and Its Application to Armor Stones and Blocks of Breakwaters

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1. INTRODUCTION

Reliability design considering probabilistic nature is quite suitable for coastal structures because waves are of irregular nature and wave actions fluctuate. However, solely considering the probability of failure is considered insufficient, as deformation (damage level) should also be taken into account.

This paper discusses performance design as an advanced design methodology for maritime structures, focusing on deformation-based reliability design of armor stones and blocks of breakwaters. The stability performance of breakwater armors is specifically considered by describing the design criteria (allowable limits) of damage level with respect to different design levels including probabilistic aspects. The accumulated damage during a lifetime is also discussed.

Fig. 1 Cross sections of breakwaters considered

Figure 1 shows the cross section of a rubble mound breakwaters and a caisson breakwater (horizontally composite breakwater), which are considered to discuss the performance design in this paper. The rubble mound breakwater has armor layers with stones while the caisson breakwater has concrete blocks to dissipate the wave energy. The stability of the armor stones and concrete blocks is the most important

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aspect in the design of the respective breakwaters.

The necessary mass of the armor stones can be determined by the Van der Meer formula (1988), while the mass of wave-dissipating concrete blocks (tetra pods) by the Takahashi-Hanzawa formula (1998). It should be noted that the design formulas are for the evaluation of the stability number as a function of deformation (damage level S/relative damage No). Therefore, the deformation can be evaluated from the formulas if the mass is given.

2. PERFORMANCE DESIGN

2.1 Definition of Performance Design

While performance design started in Europe in 1960's, it became popular in the United States following the 1994 Northrige earthquake disaster. Stability performance of buildings and civil engineering structures is assessed in the performance design (SEACO,1995).

Intensive discussions on the performance design for coastal structures were made during the International Workshop on Advanced Design of Maritime Structures in the 21st century (Takahashi, 2001; Burcharth, 2001). Although the design technology for coastal structures has seen great advancements during the 20th century, it was concluded that the design technology should be integrated to meet higher level of society's demand in the 21st century.

The concept of performance design is new and fluid, which allows researchers and engineers to create an integrated design framework for its development. Performance design can be considered as a design process that systematically and clearly defines performance requirements and respective performance evaluation methods. In other words, performance design allows the performance of a structure to be explicitly and concretely described.

The performance design was started to describe the stability performance, and was easily extended to de-scribe the functional performance. In addition, the performance related to landscape, ecosystem, amenity, lifetime durability (deterioration/maintenance) etc. should be considered, although they are not discussed in this paper.

2.2 Performance Matrix for the Performance Design

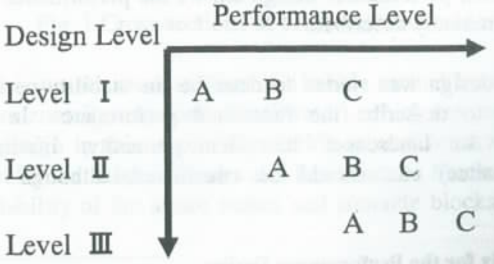
The performance design is based on a performance matrix, as shown in Table 1. The horizontal axis is the performance level, while the vertical axis is the design level. Letters A, B, and C in the table denote the importance of the structure, ie., A is critical, B is ordinary and C lesser degree. Using the performance matrix, the demand performance of a structure corresponding to the respective design levels can be explicitly indicated in consideration of importance of the structure.

The performance matrix is an essential tool for the performance design. To clearly show the performance of the structure, single design level is not enough, and the performance for different design levels should be quantitatively indicated. For example, the stability performance for different levels of storms should be evaluated. Especially the performance against an extreme storm exceeding the conventional design level is essential to design the coastal structures.

The multiple performance levels should be defined in the performance matrix. For the stability performance, the performance level is defined by four limit states; namely, serviceability limit, repairable limit, near collapse (ultimate) limit, and collapse limit, corresponding to the extent of deformation. Serviceability limit and ultimate limit are defined in the current limit states design, whereas we added the other two limit states to more quantitatively describe the change of performance. That is, the collapse limit state is defined as extremely large damage such that the inner mound is eroded significantly, while the repairable limit state is deformation that is repaired relatively easily.

The necessity of such a performance matrix had already been noticed by the experts of coastal engineering. A similar table was already presented by van der Meer (1988), since the damage of armor stones was intensively discussed by van der Meer. It should be noted that the performance matrix is a key tool to be used systematically in the performance design. The performance matrix is necessary not just for stability performance but also for all the performance of structures in the performance design. However, the stability performance matrix is relatively easily understood by the engineers in this field and the actual use will surely lead to the practical development of the performance design.

Table 1 Conceptual figure of performance matrix



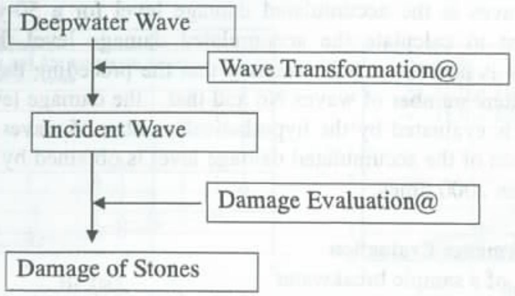
3 . PERFORMANCE DESIGN FOR ARMOR STONES OF RUBBLE MOUND BREAKWATER

3.1 Procedures to Evaluate Stability Performance

a) Deterministic value for a deepwater wave with recurrence interval

Table 2 shows the flow of the calculation method of damage level of armor stones due to a deepwater wave with particular recurrence interval. After specifying the deepwater wave, the incident wave is calculated by the wave transformation method (Goda 1985). The van der Meer Formula is used to evaluate the damage level S of armor stones due to the incident wave. The damage level S is defined by the erosion area Ac around still-water level and nominal diameter of the stones D (van der Meer, 1988).

Table 2 Flowchart for calculating damage level



b) Probabilistic value for a deepwater wave of a recurrence interval

Even for a fixed deepwater wave condition, resultant damage of stones usually fluctuates due to the probabilistic nature of propagating waves and the response of the armor stones. To obtain the probabilistic damage level for a given deepwater wave, fluctuation of the items denoted by @ should be considered. Table 3 shows parameters considered to reflect probabilistic nature in the present calculations and indicates bias of mean values and standard deviation (variance) of the probability distribution.

Table 3 Estimation error for design parameters

Design parameters	Bias of mean value	Variance
Wave transformation	0.	0.1
Size of stones	0.	0.1
Mass density of stones	0.	0.05
Deepwater wave	0.	0.1
Tidal level	0.	0.1

The Monte Carlo simulation allows calculating the probability distribution of the damage level, with the calculation being repeated more than 2000 times from wave transformation to determination of the damage level for a fixed deepwater wave condition. From the probability distribution, the mean and 5% exceedance value are selected to represent the calculated distribution

c) Accumulated value during lifetime

To obtain the accumulated damage level during breakwater lifetime (50 yr), one needs to consider the probabilistic nature of the deepwater wave and tidal level. The Weibull distribution with k = 2.0 is assumed as the extreme wave distribution with estimation error of 10% standard deviation. The tidal level is assumed as a triangle distribution between the L.W.L. and H.W.L. with error of 10% standard deviation.

A total of 50 deepwater waves are sampled and the damage level is evaluated by the Monte Carlo simulation using the procedure in Table 2. Total damage level due to the 50 deepwater waves is the accumulated damage level for a 50-yr lifetime. It should be noted that to calculate the accumulated damage level the equivalent number Nq of waves is necessary. It is assumed that the preceding damage level is caused by the equivalent number of waves Nq and that the damage level caused by the following storm is evaluated by the hypothetical number of waves (N-Nq). The probability distribution of the accumulated damage level is obtained by repeating the calculations more than 2000 times.

3.2 Example of Performance Evaluation

(1) Design condition of a sample breakwater

Stability performance of armor stones is illustrated here using a sample cross section of rubble mound breakwater. The conventional design condition of the breakwater is that the water depth h= 10m, wave period T=10.1s, deepwater wave height H_o=4.76m, wave height at the breakwater $H_{1/3}$=4.65m (Number of waves N=1000). The designed mass of the stones M=11.7t when the armor slope is 1:2, the designed damage level S=2 and armor notional permeability factor P=0.4.

(2) Deepwater wave and damage level (deterministic value)

Figure 2 shows damage level of armor stones of rubble mound breakwater produced by deepwater waves of different recurrence intervals, where the deterministic value of the damage level denoted by □ is 2 when the design wave with a 50-yr recurrence interval attacks, as designed.

Note that as the deepwater wave height increases, the damage level increases gradually; i.e., the damage level for a 500-year recurrence interval is 3.9, while the damage level for a 5-year recurrence interval is 0.9.

(3) Deepwater wave and damage level (probabilistic value)

Figure 2 also shows the damage level due to deepwater waves obtained from the Monte Carlo simulation that included fluctuation of waves and block damage, where the mean (◇) and 5 % exceedance (↑) values of relative damage are indicated. Due to the probabilistic nature, i.e., the occurrence of larger incident wave height and larger damage, even the mean value of the relative damage is greater than the deterministic values. In fact, the 5% exceedance value is much larger the mean value. For example, for a wave with a 50-yr recurrence interval the mean value of the damage level is 2.9 and the 5% exceedance value is 8.7, whereas for a wave with a 500-yr recurrence interval the values are 5.5 and 16.0, respectively. Obviously then, the probabilistic nature must be considered.

(4) Damage level for different stone masses

Figure 3 shows the mean value of damage level for different stone masses. When the mass is 1.2 times the design mass, the mean value of damage level for a 50-yr recurrence interval is 2.13, 75% of the mean value for the design mass. When the mass is 1.5 times the design mass, the mean value of damage level is 1.47, a half of the mean value for the design mass.

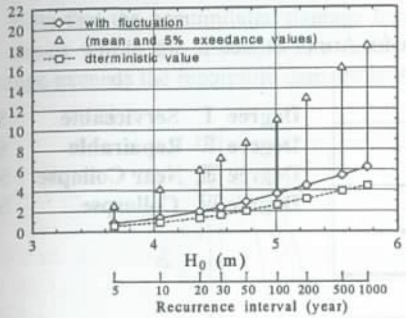


Fig. 2 Deepwater wave height vs. damage level

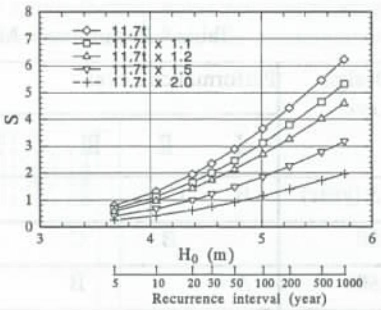


Fig. 3 Damage level for different stone masses

3.3 Performance Matrix for Armor Stones

Table 4 shows a so-called performance matrix for armor stones. The vertical axis is the design level corresponding to waves with four different recurrence intervals, while the horizontal axis is the performance level defined by four limit states corresponding to the extent of deformation (damage). These limit states are defined by deformation, being the mean value of the relative damage in this case.

Figure 4 shows the probability of exceedance of the occurrence of the deepwater wave over a 50-yr lifetime (design working time) vs. the recurrence interval of the deepwater wave. Since the estimation error in the Weibull distribution is considered to be 0.1 (variance), the probability of exceedance for the wave of a 50-yr recurrence interval is > 80%, being high compared to the conventional value of 63%. Even for the wave of a 500-yr recurrence interval the exceedance is still high, nearly 30% and for the wave of 5000-yr recurrence interval the probability is still nearly 10%. Considering the occurrence probability, the design level should be selected. The performance matrix in Table 4 includes a very extreme condition of 5000-yr recurrence interval.

Table 5 shows relation between the value of damage level and the actual extent of damage given by van der Meer(1988). For the case of 1:2 slope, the initial damage is when S=2 and the actual failure (exposure of the filter layer) is defined by S=8. In the performance matrix the serviceable limit is defined by S=2, Repairable S=4, Near collapse S=6 and collapse S=8, considering the definition in Table 5. For the structure of ordinary importance B, required stability performance is S=2 for a wave of 50-yr recurrence interval, and S=8 for 5000-yr recurrence interval in the performance matrix.

The values indicated here are so-called design criteria or allowable limits and are tentatively determined. Using the figure like Fig.3 with Table 4 we can determine the necessary mass of the armor stones.

Table 4 Performance Matrix for Armor Stones

Design Level	Performance Level			
	I	Ⅱ	Ⅲ	Ⅳ
5 (year)	B	C		
50		B	C	
500	A		B	C
5000		A		B

Degree Ⅰ	**Serviceable**	S=2
Degree Ⅱ	**Repairable**	S=4
Degree Ⅲ	**Near Collapse**	S=6
Degree Ⅳ	**Collaspse**	S=8

Table 5 Damage level S vs. failure level

Slope	Initial Damage	Intermediate Damage	Failure
1:1.5	2	3-5	8
1:2	2	4-6	8
1:3	2	6-9	12
1:4	3	8-12	17
1:6	3	8-12	17

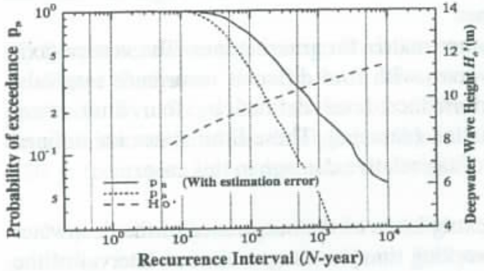


Figure 4 Probability of exceedance for a wave with various recurrence intervals over a 50-yr lifetime.

3.4 Lifetime Stability Performance

Figure 5 shows the probability of exceedance of accumulated damage level over a 50-yr breakwater lifetime for different stone masses. The mean value of the accumulated damage level, which we call the "expected damage level," is 8.2 for the stones of the design mass. The probability of exceedance for an accumulated relative damage of 16.6 is 5%.

The value of the expected damage level of 8.2 is 4 times the design value and corresponds to the mean damage level for the wave of 5000-year recurrence interval. It can be said that the damage is accumulated significantly even by relatively small storms during the 50-year lifetime. However, this can be due to the characteristics of the van der Meer Formula and the current calculation procedure to accumulate the damage in this paper. It should be noted that the square root function of N gives the relatively large damage level compared with the experimental results. Some modification was suggested by van der Meer (1988) and Melby and Kobayashi (1998) proposed a damage formula including the progress of the damage.

Note that the accumulated damage level is a hypothetical value not considering the repair within the lifetime. The repair should be made before the accumulated damage exceeds the repairable damage level.

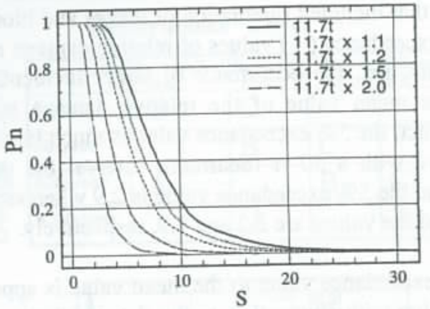


Fig.5 Probability of exceedance of accumulated damage level

4. PERFORMANCE DESIGN FOR CONCRETE BLOCKS COVERING VERTICAL WALL

4.1 Example of Performance Evaluation

(1) Design condition of a sample breakwater

The almost the same procedures and the conditions are used to evaluate the stability performance for concrete blocks covering vertical wall. The designed mass of the blocks M=10.3t when the armor slope is 1:4/3, and the designed relative damage No=0.3. The relative damage is the number of blocks per unit length(nominal diameter of the block) in the direction of breakwater alignment, which can be evaluated by the Takahashi-Hanzawa formula (Takahashi et. al. 1998).

(2) Deepwater wave and relative damage (deterministic value)

Figure 6 shows relative damage of concrete blocks covering the caisson produced by deepwater waves of different recurrence intervals, where the deterministic value of the relative damage denoted by □ is 0.3 when the design wave with a 50-yr recurrence interval attacks, as designed

Note that as the deepwater wave height increases, the relative damage increases gradually; i.e., the relative damage for a 500-year recurrence interval is 1.3, while the relative damage for a 5-year recurrence interval is almost 0.

The damage level of armor stones shown in Fig.2 indicate the damage starts from relatively small wave and gradually increase, while the relative damage of concrete blocks in Fig. 6 does not appear when the wave is small and increases relatively significantly with the increase of wave height. This trend of the damage is due to the characteristics of the respective formulas (the van der Meer Formula and Takahashi-Hanzawa Formula).

(3) Deepwater wave and relative damage (probabilistic value)

Figure 6 also shows the relative damage due to deepwater waves obtained from the Monte Carlo simulation that included fluctuation of waves and block damage, where the mean (◇) and 5 % exceedance (↑) values of relative damage are indicated. Due to the probabilistic nature, i.e., the occurrence of larger incident wave height and larger damage, even the mean value of the relative damage is greater than the deterministic values. In fact, the 5% exceedance value is much larger the mean value. For example, for a wave with a 50-yr recurrence interval the mean value of the relative damage is 0.7 and the 5% exceedance value is 2.9 whereas for a wave with a 500-yr recurrence interval the values are 2.2 and 7.9, respectively.

The ratio of the 5% exceedance value to the mean value is approximately 4, and the ratio of the mean value with fluctuation to the deterministic value is around 2. Those ratios for the damage level of armor stones is 3 and 1.5 as shown in Fig.2, smaller than 4 and 2 respectively. This trend comes also from the characteristics of the damage formulas.

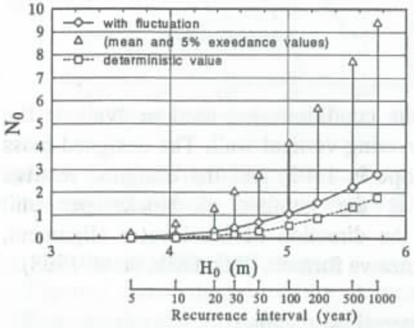


Fig. 6 Relative damage vs. recurrence interval

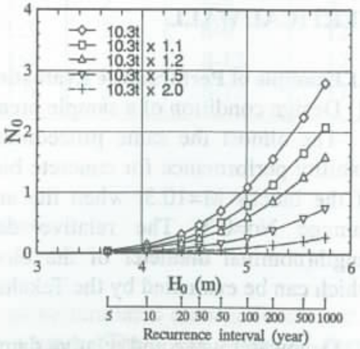


Fig. 7 Relative damage for different block mass

(4) Damage level for different block masses

Figure 7 shows the mean value of relative damage for different block masses. When the block mass is 1.2 times the design mass, the mean value of relative damage is 0.36, a half of the mean value for the design mass. It can de found that compared with the damage of armor stones, the increase of mass is more effective.

4.2 Performance Matrix for Concrete Blocks

Table 6 shows a performance matrix for the concrete blocks covering the vertical wall. The vertical axis is the design level corresponding to waves with four different recurrence intervals, while the horizontal axis is the performance level defined by four limit states corresponding to the extent of damage as already discussed. These limit states are defined by deformation, being the mean value of the relative damage in this case.

Table 6 Performance matrix

Performance Level

Design Level

Limit states	Serviceability (0.3)	Repairable (0.9)	Ultimate (1.8)	Collapse (3.0)
5-year	B	C		
50-year		B	C	
500-year	A		B	C
5000-year		A		B

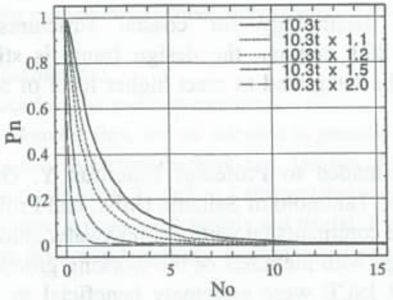


Fig.8 Probability of exceedance of accumulated relative damage

4.3 Lifetime Stability Performance

Figure 8 shows the probability of exceedance of accumulated relative damage distance over a 50-yr breakwater lifetime for different block masses. The mean value of the accumulated relative damage, which we call the "expected relative damage," is 2.24 for the blocks of the design mass. The probability of exceedance for an accumulated relative damage of 6.46 is 5%..

The value of the expected relative damage of 2.24 is more than 7 times the design value and corresponds to the mean damage level for the wave of 500-year recurrence interval. It can be said that the damage is accumulated significantly during the

50-year lifetime. For the concrete blocks, those values are 4 times and 5000-year. This is due to the difference of the characteristics of the formulas.

5 . CONCLUDING REMARKS

Nowadays it is crucial to obtain the understanding of the general public regarding the construction of coastal structures. Designs must incorporate accountability, and the performance of the facility must be explicitly and clearly explained. The best way for the citizens to understand the performance is to see what is actually happened when the storm attacks. To have better understanding from citizens, the performance should be described vividly like a scenario.

The actual failure is a prototype performance evaluation of the structure against the occurred storm, and the intensive investigation on the disaster usually done after the disaster is like a writing a scenario to describe what was happed by the storm. If such a scenario is made in the design stage, the stability performance can be understood very clearly by citizens. The performance design should include many scenarios for different occasions ie., different levels of storms. The design with many scenarios is actually the performance design, and therefore the performance design can be said as a scenario-based design.

In addition to having scenarios related to stability and functional performance, the future performance design should include the scenarios on durability and environmental aspects including amenity, landscape, and ecology.

Although the design technology for coastal structures has seen great advancements during the 20th century, the design frame is still unchanged. The design technology should be integrated to meet higher level of society's demand in the 21st century.

Sincere gratitude is extended to Professor Emeritus Y. Goda of Yokohama National Univ., Professor K. Tanimoto of Saitama Univ., and Professor T. Takayama of Kyoto Univ. for valuable comments on vertical breakwater studies. Discussions on the new design methodology with members of the working group within the Coastal Engineering Committee of JSCE were extremely beneficial in writing this paper; especially those with Professor T. Yasuda of Gifu Univ. and Professor S. Sato of Tokyo Univ.

DISCUSSIONS

After presenting the paper at Coastal Structures 2003 (Portland), I received a discussion letter from Prof. Hans Burcharth of Aalborg University, who is as all the coastal engineers knows one of the most experienced and distinguished coastal engineers in the world. Since it is very important for this paper, I like to include here the discussion with his comments and the reply from the first author.

Discussion from Prof. H. Burcharth:

I have studied your Portland Coastal Structures ´03 paper and have the following comments;

In section 2.1 you claim that "Discussion on the performance design for coastal structures were first made during the Int. Workshop on Advanced Design of Maritime Structures in the 21´st century" and you give the reference (Takahashi, 2001).

This is not correct and should be corrected. Performance based design has been used in coastal engineering like in other types of structural engineering for many – many years. For example it is very common to design a beam for a specific maximum deformation when exposed to service loading, and to design an armour layer for 5% displaced blocks for a given occurrence of waves within lifetime. Not always have the performance limits been defined on the basis of economic optimization and consequent probabilistic analyses, but design based on performance has been done for years.

The more advanced performance design has been discussed in many of my paper, f. ex. In my Keynote address in the Int. Workshop at PHRI in 2001 and in my Keynote Address for Coastal Structures ´99. In these papers I do not use the word Performance explicitly, simply because performance based design is a trivial term because design are normally based on performance. The important point is to determine the optimum probability of failures (defined as degrees of damage – i.e. the performance - for each failure mode and design limits state) within structure life time for each type of structure (f. ex. Classified as suggested in my Coastal Structures ´99 paper)

I hope this explanation can help clarifying the historic situation related to performance based design.

I have started the work in new PIANC working group on determination of optimum design safety levels for breakwaters following the Safety Class – Limit State – Failure Probability scheme outlined in my papers, and in fact already used in some standards for civil engineering structures as the basis for calibration of partial safety factors.

Discussion from S. Takahashi:

Thank you very much for your precious comments.

1) I do not intend to claim the first, and we intended to promote the performance design in the coastal engineering field by having the workshop. The aim of this paper is also the same, not to describe the historic situation. Therefore, I changed that part considering your suggestion.
2) The term of performance design is definitely **not trivial**. It represents the new direction of the coastal structure designs as frequently stated in my paper and the panel discussions in the workshop.
@ The performance design is toward the citizens (not just toward professional engineers):
Citizens need to clearly understand various performances of all the lifetime of the structure.
@ The performance design is for young researchers and engineers to come:
The coastal structure design had developed greatly in the 20th century. However, it is not enough for the demands from citizens in this century. Young researchers and engineers are encouraged to develop the new design not just by extending the existing old design but by cultivating an entirely new design scheme under the new name such as performance design.
3) The design based on the partial safety coefficients is a traditional design method and very convenient for engineers. However, it is like a black box for citizens impossible to understand and is therefore, not appropriate for the performance design.
4) Under the PIANC, I know a lot of innovative working groups being organized to cultivate new fields and new concepts. I feel we need a creative working group for coastal engineering to develop new design schemes for new century. We have to provide creative fields for

young researchers and engineers to be active and innovative.

5) If possible, I like to have a chance to discuss above more with engineers and researchers in the next coastal structure conference or in some other meetings.

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