Flood Risk Management: Research and Practice – Samuels et al. (eds)

Calculation of fragility curves for flood defence assets

J.W. van der Meer
Van der Meer Consulting, Heerenveen, The Netherlands

W.L.A. ter Horst
Infram BV, Marknesse, The Netherlands
Delft University of Technology, Section of Hydraulic Engineering, Delft, The Netherlands

E.H. van Velzen
Rijkswaterstaat, Centre for Water Management, Lelystad, The Netherlands

ABSTRACT: Aim of this paper is to give not only the theoretical background of calculation of fragility curves, but also practical applications. The theoretical background is given first, which leads to the description of load and strength distributions. There are large differences in load situations, such as high river discharges (water level very close to the crest of the dike, hardly any waves) and severe storms (crest much higher than the surge level, large wave attack). These are elaborated with an example for a river dike.

A case study was undertaken for a large sea dike and the elaboration was concentrated on failure due to wave overtopping. Wave overtopping is well-known, but good knowledge about the resistance to overtopping waves is still lacking. Based on recent tests, however, a preliminary conclusion is made for the critical overtopping discharge to be 30 l/s per m. The paper then gives the implications, very well shown by fragility curves, of a design made for 1 l/s per m overtopping and the inherent safety if the dike only fails for 30 l/s per m.

1 INTRODUCTION

Safety assessments of flood defence assets are increasingly performed with the technique of structural reliability. All parameters, load parameters (hydraulic boundary conditions) and strength parameters (dike characteristics), are taken into account and expressed as stochastic variables. One of these structural reliability methods is to calculate the failure probability (P_f) of a flood defence, given a certain water level. Assembling the failure probabilities for several water levels constructs a fragility curve. The paper will first describe the theoretical background.

The use of fragility curves for flood defence assets makes it possible to give a fair estimate of what failure mechanism will be dominant for a certain extreme water level, including waves. The main failure mechanisms are wave overtopping, stability of the seaward revetment, piping and instability of the inner slope. An example will be given for a river dike and the case study, described in this paper, considers a large sea dike. In this case study the difference between design and failure will be made clear, using fragility curves.

2 PROBABILISTIC DESCRIPTION OF A FRAGILITY CURVE

2.1 *Failure probability and fragility*

The fragility of a dike section can be defined as the failure probability conditional on a specific loading (Casciati and Faravelli, 1991 and Buijs et al, 2007). The failure probability can come from a combination of various failure mechanisms and the loading may be a water level only, or water level (storm surge) and high waves. This section gives a probabilistic description of a fragility curve of a dike section.

The general description of the failure probability of a dike section can be described as (Kuijper & Vrijling, 1998):

$$P_f = P(Z \le 0) = P(R \le S) = \iint_{Z \le 0} f_{RS}(r,s)\,dr\,ds \qquad (1)$$

where Z = R−S, P_f = failure probability, Z = limit state function, R = strength parameter, S = load parameter and f_{RS} = joint probability density function of R and S.

Assuming R and S to be independent the failure probability can be described by the following integral, where s is a certain value of the load S:

$$P_f = \int_{s=-\infty}^{s=\infty} \int_{r=-\infty}^{s} f_s(s) \cdot f_R(r) drd \tag{2}$$

which can be written as:

$$\begin{aligned} P_f &= \int_{s=-\infty}^{s=\infty} f_s(s) \cdot \int_{r=-\infty}^{s} f_R(r) drds \\ &= \int_{s=-\infty}^{s=\infty} f_s(s) \cdot F_R(s) ds \end{aligned} \tag{3}$$

where $f_S(s)$ = probability density function (pdf) for random variables of load S and F_R(s) = cumulative density function (cdf) which returns the failure probability given the value s of a certain load S.

Assuming the water level h_w to be the only load parameter, as is often the case in situations with high discharges in rivers, the expression for the failure probability can be described as:

$$P_f = \int_{s=-\infty}^{s=\infty} f_{hw}(h_w) \cdot F_R(h_w) dh_w \tag{4}$$

where $f_{hw}(h_w)$ = the probability distribution function of the water level and $F_R(h_w)$ = the cumulative distribution function of the strength given a certain water level h_w.

The cumulative distribution function of the strength of the dike section gives the relation between the load and the failure probability given this load and is simply called the fragility curve of the dike section, see Figure 1.

Actually, the $F_R(h_w)$-term of Equation 4 can be seen as the fragility curve. It gives the cumulative distribution function of the strength (R) of the dike section

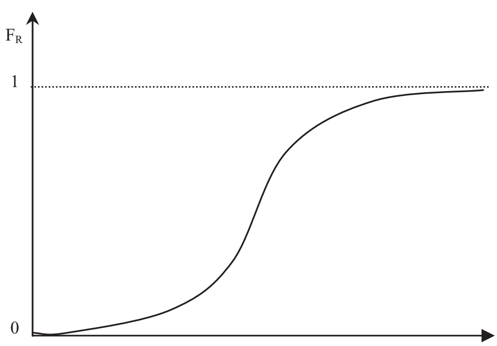

Figure 1. Fragility curve.

given a certain load condition. The strength can still be described by various failure mechanisms.

2.2 *Load distributions*

Depending on the actual situation various load parameters as well as distributions are possible. One situation may be a high river discharge, where the water level is the determining parameter. Any waves will probably be small and also independent from the probability of a high river discharge. In many of these cases the water level will be close to the crest of the dike.

A useful application of fragility curves is during these flood waves (high river discharges). With the threat of a possible flooding it is valuable to have information on the failure probability of a dike section given a certain expected water level. Therefore water level predictions can be helpful. In (Ter Horst, 2005) water level predictions and their distribution functions are discussed more explicitly.

An other situation is where a storm gives high storm surges in combination with large waves. Now the water level and wave conditions are to some extent dependent. In these situations the crest is often well above the water level, due to the large waves which give also large run-up or extensive overtopping.

Both situations may be possible where a river flows into the sea or ocean. In such a case a high river discharge as well as a large storm may attack the dike, but both extreme situations are independent (it is not likely that a very extreme storm coincides with a very extreme river discharge).

Other situations may be reservoir dams, where water level and waves have a different dependency than for rivers or the sea.

A commonly used method is to describe the relation between the value of a certain load parameter X and the return period T(Vrouwenvelder, 2003). In general this relation between X and T can be written as:

$$X = b \cdot \log(T) + a \tag{5}$$

where X = the value of the load parameter, b = decimetrical height, T = return period corresponding with the value of X and a is the reference level. The decimetrical height is equal to the change in water level corresponding with a reduction of the ex-ceedance probability with a factor 10.

The event of X crossing a certain threshold level can be seen as a Poisson process. This means several threshold crossings in time can be seen as independent events. The relation between the return period T and the exceedance probability P can be written as:

$$P[X > X^*] = 1 - \exp^{\frac{1}{T}} \tag{6}$$

where X^* = the threshold level of the load parameter X. Combining (5) and (6) leads to the following relation between the exceedance probability P and the level X of load parameter:

$$P[X > X^*] = 1 - e^{-e^{[\frac{a-X^*}{b}]}} \tag{7}$$

Formula 7 presents a general expression for the event that the load exceeds a certain threshold value within a certain period. The considered period is regularly a year and the corresponding extreme load is called a yearly maximum. Formula 7 is therefore an extreme value distribution (Gumbell distribution).

Formula 7 can be applied on various load parameters, i.e. river discharge, wave height, sea water level and wind speed.

2.3 *Failure mechanisms*

The responses to various loads may be different, depending on the geometry of the dike and its surroundings. Many failure mechanisms can be considered, see the Floodsite Task 4 report "Failure Mechanisms for Flood Defence Structures" (Floodsite, 2007). Some mechanisms have better been described than others and also some mechanisms are more important than others, depending on the actual situation. The fragility curve of a dike section is a combination of all fragility curves for the individual failure mechanisms.

In this paper a few failure mechanisms have been considered as essential and one will be elaborated more in depth: strength against wave overtopping. The most important failure mechanisms are often:

– Instability of outer protection (revetments).
– Resistance against wave overtopping
– Piping
– Instability of inner slope

An example of three individual fragility curves for an actual case, were described by Ter Horst (2005), see Figure 2. A river dike is considered with the water level close to the crest and with fairly small wave attack.

Figure 2 shows in a nice way how different individual fragility curves contribute to the total fragility curve for the dike section. In the example, the dike has a crest level at about 18 m +NAP (NAP is a reference level). As long as the water level is more than about 0.5 m lower than the crest level, wave overtopping will not occur and, therefore, wave overtopping does not contribute to the failure probability. It also shows that in this situation the water level is very close to the crest of the dike. As soon as the water level comes close to the crest, the failure probability for wave overtopping increases rapidly and determines then the total failure probability.

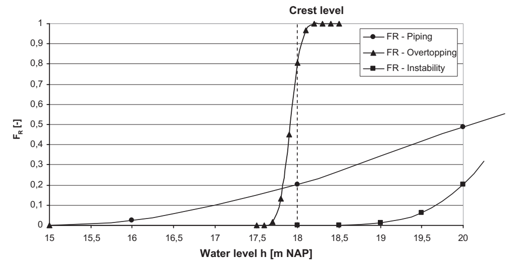

Figure 2. Fragility curves for three failure mechanisms as function of the water level. Situation of high river discharge.

The piping mechanism gives a fragility curve which gradually increases with increasing water level. But actual probabilities start already for 15 m +NAP, which is a water level 3 m below the crest level. Although the probability of failure does not exceed 20%, given the specific water level, it is clear that for water levels lower than about 17.5 m +NAP piping is the governing failure mode.

In the given situation of Figure 2 the mechanism of infiltration of water through the dike, causing instability of the inner slope, is not governing at all. The failure probability is still zero if the water level exceeds the crest height.

Figure 2 gives three individual fragility curves. The actual fragility curve for the dike section is the left boundary, given by all the curves. This means first the curve for piping up to about 20% and then the curve for overtopping up to 100% (or 1 in the graph).

3 DESCRIPTION OF FRAGILITY CURVES BY FAILURE MECHANISMS

3.1 *Description of overtopping*

Wave overtopping has been a favourite research item in the last decade and has resulted in the new Overtopping Manual (2007). Wave overtopping depend on a lot of parameters, like wave height, wave period, angle of wave attack and structural parameters like slope angles, a berm, roughness on the slope, relative crest height, etc. The Overtopping Manual gives formulae, but also calculation tools like PC-OVERTOPPING and the neural network prediction from CLASH.

The most simple equation for wave overtopping, assuming a straight smooth slope, is:

$$\frac{q_0}{\sqrt{g \cdot H_{m0}^3}} = \frac{0.067}{\sqrt{\tan\alpha}} \xi_{m-1,0} \cdot \exp\left(-4.3 \frac{R_C}{\xi_{m-1,0} \cdot H_{m0}}\right) \tag{8}$$

with a maximum of:

$$\frac{q_0}{\sqrt{g \cdot H_{m0}^3}} = 0.2 \cdot \exp\left(-2.3 \frac{R_C}{H_{m0}}\right) \tag{9}$$

where the breaker parameter ξ_m-1,0 is given by:

$$\xi_{m-1,0} = \tan\alpha / \sqrt{S_{m-1,0}} \tag{10}$$

where q_o = the average wave overtopping discharge, g = the acceleration due to gravity, H_{m0} = significant wave height at toe of dike, $s_{m-1,0}$ = wave steepness, *Tm-1.0* = spectral wave period at toe of dike, *tan* α = slope angle and R_c = crest freeboard above still water line.

Overtopping equations describe the hydraulic behaviour of waves on a dike, they do not yet describe the strength of the dike. But the easiest way to do that is to assume a certain critical overtopping discharge, q_c, which is assumed to give "failure" of the structure (erosion or sliding of inner slope, leading to a breach).

3.2 *Limit state function*

Equations 8–10 can be used to calculate the average overtopping, given the structural geometry and the wave boundary condition. It describes only the load on the inner slope of the dike. The limit state function based on the equations 8–10 can be expressed as:

$$Z = m_c \cdot q_c - m_o \cdot q_o \tag{11}$$

where m_c = the model factor for critical overtopping discharge, q_c, and m_o = the model factor for the actual overtopping discharge. The model factor m_o gives the reliability of the prediction of overtopping and has been described in the Overtopping Manual (2007) or TAW (2002). In this paper q_c is set at a certain value, although modelling exists to calculate a value. This modelling, however, is only based on overflow of earthen dams, not on wave overtopping. This paper will give the background for the decision on a value for q_c.

4 CASE STUDIES: SEA DIKE AND LAKE DIKE

4.1 *Fragility curves for a high sea dike*

Calculations were performed for an existing sea dike with direct wave attack from the North Sea, but cross-sections and parameters were schematized in such a way that results do not necessarily represent the actual dike. This is why the location of the case study has not been mentioned.

The schematized cross-section is given in Figure 3. The reference level NAP is more or less mean sea level. The dike crest is about 12 m above this level and represents indeed a high dike. The toe of the dike is situated just below NAP and in front of the dike a rather steep foreshore of about 1:30 is present. The lower slope of the dike has been schematized as a slope 1:8 (in reality it is a 1:4 down slope with a berm) and the upper slope is about 1:3.

Calculations have been done for overtopping and piping. For overtopping various "critical" overtopping discharges, q_c, were used: 1; 10; 30 and 50 l/s per m. The final results are given in Figure 3, which shows the individual fragility curves, depending on the water level (=storm surge + tide). Two more (vertical) lines have been given, the design water level or water level to be used for safety assessment at 5 m +NAP and the crest level at 12 m + NAP.

A few conclusions can be drawn from Figure 4. The water level for safety assessment at 5 m +NAP is reached for a return period of 10,000 years, or in an other way, has a probably of occurrence of 10–4 per year. This is a very extreme event, but the consequences of a flooding in the Netherlands are also extreme. For this extreme event the difference between the crest level and the water level or storm surge level is still about 7 m. This is completely different from river dikes, where the water level may come very close (within half a metre) of the crest. But the wave heights are of course fairly large, for the 10–4-event about 3 m at the toe of the dike (and over 10 m a few kilometers offshore).

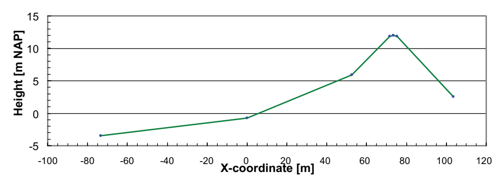

Figure 3. Schematized cross-section of the sea dike in the case study.

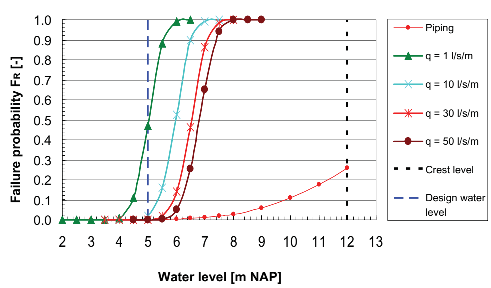

Figure 4. Fragility curves for piping and overtopping.

A second conclusion is that in this case piping is not a governing failure mechanism. Independent of the choice of "critical" overtopping discharge, the failure probability for piping is always smaller. Stability of the seaward protection (the revetment from the toe of the dike up to the crests) has not been calculated in the present case study. This leaves only wave overtopping as the governing failure mechanism. The dike has been designed for 1 l/s per m wave overtopping for the 10^{-4}-event. Figure 3 shows a probability of failure, or rather a probability of occurrence, of about 50%. It is assumed that each dike can cope with 1 l/s per m overtopping, as long grass is present on the crest and inner slope. Figure 3 shows that this discharge has a probability of occurrence of 10% if the water level is 0.5 m lower and of about 90% if the water level is 0.5 m higher than the 10^{-4}-level of 5 m +NAP.

The fragility curves for larger overtopping distributions are more or less parallel to the 1 l/s per m curve, but are shifted to the right in Figure 3. It is very unlikely that 10 l/s per m overtopping will occur for the water level of 5 m +NAP (10^{-4}-event). For a probability of occurrence of about 50% this overtopping discharge needs a water level that is about 1 m higher. For the 50 l/s per m discharge it needs even a 2 m higher water level.

Figure 4 shows how different overtopping criteria are dependent on the water level, but it gives no information on the probability of occurrence of that water level. However, the water level and wave statistics are known and Figure 4 can also be transformed into a graph where the horizontal axis gives the return period of the events. This results in Fig. 5.

Figure 5 shows that 50% probability on 1 l/s per m overtopping has a return period of about 10,000 years (the design event). The graphs shows also that 10 l/s per m overtopping (with 50% probability) will occur once in 3.10^5 years. For 30 l/s per m this increases to once in 2.10^6. This is of course a very rare event.

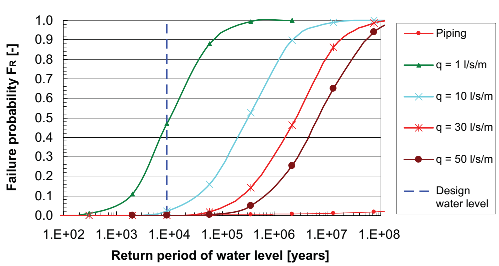

Figure 5. Fragility curves as a function of the return period of the water level.

4.2 *Erosion by wave overtopping*

Fragility curves for various critical overtopping discharges give some insight, but the most important question is: what critical overtopping discharge will lead to real failure (breaching) of the dike? Two mechanisms may lead to failure due to wave overtopping. The first is infiltration of overtopping water into the dike and eventually sliding of the inner slope. The second is erosion of the cover layer of clay and grass by overtopping waves, followed by erosion of the inner slope (clay or clay layer on sand core).

The first mechanism, infiltration and sliding, can only occur if the inner slope is quite steep. For this reason most dike designs after the flood of 1953 got a 1:3 inner slope in the Netherlands. It is assumed that such a slope will not slide due to infiltration of water. But if a steeper slope is present, already 1 l/s per m overtopping would be enough to give sufficient infiltration of water.

This means that for steep inner slopes (steeper than 1:3 or may be 1:2.5) the critical overtopping discharge is already 1 l/s per m. The dike in the case study has an inner slope of 1:3 and this means that infiltration and sliding is not a governing failure mechanism. Only erosion by overtopping remains.

Till a few years ago hardly anything was known about resistance of inner slopes of dikes with grass against wave overtopping. But in the beginning of 2007 and 2008 innovative destructive tests have been performed for various dike sections. In 2006 the wave overtopping simulator has been constructed, see Van der Meer et al., 2006. The basic idea is that a constant discharge is pumped into a box on top of a dike and then the pumped volume is released from time to time in such a way on the inner slope that it simulates overtopping waves in reality. Figure 6 gives an impression of the working of this wave overtopping simulator.

Figure 6. The wave overtopping simulator releases 22 m^3 of water over 4 m width in about 5 s. It simulates a large overtopping wave with an average discharge of 75 l/s per m.

Tests have been performed for overtopping discharges starting at 0.1 l/s per m up to 75 l/s per m. In 2007 3 dike sections have been tested, which are reported by Van der Meer et al., 2007, Akkerman et al., 2007 and in the ComCoast reports (www.comcoast.org). Early 2008 another 9 dike sections have been tested at three locations (see Figure 7) in the Netherlands. Part of the results has been given by Steendam et al., 2008, at this conference.

They come to a few preliminary conclusions and the most important one in relation to fragility curves is:

It seems unlikely that an inner slope with a clay cover topped with a grass cover (in Dutch situations) will fail due to erosion by overtopping waves with a mean discharge of 30 l/s per m or less. Future research may result in a final conclusion.

A large number of dike sections withstood 50 l/s per m and some of them even 75 l/s per m. No section failed for 30 l/s per m, which gives the basis for the preliminary conclusion. This means that the 30 l/s per m curve in Figures 4 and 5 can be considered as a minimum q_c, with respect to real erosion failure by wave overtopping. Consequences will be discussed in the next section.

Figure 7. Damage to a dike section during a test with 75 l/s per m wave overtopping.

4.3 *Fragile or robust*

The name fragility curves means that the curves say something about the fragility of the dike against extreme events. But how fragile is a dike actually, if it has been designed for 1 l/s per m overtopping?

Recently, another related topic came up in the Netherlands with respect to the relatively large and high dikes there. If already designed for a 10,000 years event, how much will it take to design it to be indestructible? Of course, in probability analysis the probability of zero does not exist, there is always an extremely small probability. For that reason "indestructible" was defined as:

"A dike which has a probability of failure (=breaching), which is practically zero. Practically zero is then 2 orders of magnitude more safe than the present standards."

For the dike in the case study this would mean that it should be strong enough to withstand a 10^{-6}-event, an event that only occurs once in a million years.

Wave conditions and water levels were determined for the 10^{-4}, 10^{-5} and 10^{-6}-events and then PC-OVERTOPPING was used to calculate the overtopping discharges. These were respectively 0.6, 4.5 and 19 l/s per m. The first condition is more or less the design condition. The 19 l/s per m overtopping discharge is still smaller than the limit of 30 l/s per m. This actually means that a design with 1 l/s per m overtopping leads to a robust and "indestructible" dike section (only with respect to erosion by overtopping).

Also the fragility curves in Figure 3 give a similar answer. The 50%-probability for 30 l/s per m in this graph gave a return period of 2.10^{-6}, which is more extreme than the 10^{-6}-event. One can say that the difference between the curves for 1 and 30 l/s per m in Figure 4 gives the safety between design and failure and that the probabilities for the 30 l/s per m curve actually say that this dike section is "indestructible" with respect to erosion by wave overtopping.

A more extreme event does not only lead to higher water levels, but also to larger waves. Another failure mechanism, not (yet) modeled in the case study, is stability of the revetment. Most stability formulae are based on the stability number $H_s/\Delta D$, where H_s = the significant wave height (at the toe of the dike), Δ = relative mass density and D = a diameter or thickness.

A larger wave height leads then linearly to a larger diameter or thickness. The increase in wave height from a 10^{-4} to a 10^{-6}-event is more or less the same increase that is required to make the revetment "indestructible". In the case study this increase was 11%. The consequence to make an "indestructible" revetment would be to increase the thickness by at least 11% and also to apply the revetment to a higher level, as the 10^{-6}-event has a higher water level.

5 CONCLUDING REMARKS

Fragility curves give information on the failure probability of a dike section given a certain load.

The relation between the load condition and the failure probability can be used for several purposes. During flood wave situations (high river discharges) water level predictions can be transformed into information on the reliability of the dike section with the help of fragility curves. Next to this, fragility curves can be helpful in determining effective measures to ensure dike safety on the long run.

When focusing on the overtopping phenomenon for a large sea dike, fragility curves give information on the failure probability of this section for storm surge levels with various return periods. In this paper these fragility curves were given for several critical overtopping discharges, from 1 to 50 l/s per m. Recent tests with the wave overtopping simulator, gave a first indication about the real critical wave overtopping for the inner slope of a dike covered with clay and grass. It was concluded that there is a large safety margin between a design value of 1 l/s per m and a critical overtopping discharge of 30 l/s per m.

REFERENCES

Akkerman, G.J., P. Bernardini, J.W. van der Meer, H. Verheij and A. van Hoven, 2007. *Field tests on sea defences subject to wave overtopping.* Proc. Coastal Structures, Venice.

Allsop N.W.H., A. Kortenhaus and M. Morris, 2007, *Failure Mechanisms for Flood Defence Structures,* Task 4 report (nr.T04-06-01) Wallingford: FloodSite.

Buijs, F.A,. J. Simm, M. Wallis and P. Sayers, 2007. *Performance and Reliability of Flood and Coastal Defences* (report FD2318/TR1). London, Defra/EA.

Casciati, F., Faravelli, L. 1991. *Fragility analysis of complex structural systems.* Taunton: Research Studies Press.

Horst, W.L.A. ter,. 2005. *The safety of dikes during flood waves (MSc-thesis).* Delft: TU Delft.

Kuijper, H.K.T., Vrijling, J.K. 1998. *Probabilistic approach and risk analysis.* Delft: TU Delft.

Overtopping Manual, 2007. Pullen T., N.W.H. Allsop, T. Bruce, A. Kortenhaus, H. Schüttrumpf and J.W. van der Meer. *EurOtop: Wave Overtopping of Sea Defences and Related Structures: Assessment Manual* (pdf download available from: www.overtopping-manual.com)

Steendam, G.J., W. de Vries, J.W. van der Meer, A. van Hoven, G. de Raat and J.Y. Frissel, 2008. *Influence of management and maintenance on erosive impact of wave overtopping on grass covered slopes of dikes.* Proc. FloodRisk, Oxford, UK.

TAW, 2002. Technical Advisory Committee on Water Defences (TAW). 2002. *Technical report on wave run-up and wave overtopping at dikes.* Delft.

Van der Meer, J.W., P. Bernardini, W. Snijders and E. Regeling, 2006. *The wave overtopping simulator.* ASCE, proc. ICCE, San Diego.

Van der Meer, J.W., P. Bernardini, G.J. Steendam, G.J. Akkerman and G.J.C.M. Hoffmans, 2007. *The wave overtopping simulator in action.* Proc. Coastal Structures, Venice.

Vrouwenvelder, A.C.W.M., Steenbergen, H.M.G.M. 2003. *Theoriehandleiding PC Ring V3.0—deel A, B en C (in Dutch).* Delft: TNO.