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Stability of Breakwater Armour Layers — Design Formulae

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ABSTRACT

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New practical design formulae have been developed which describe the stability of rubble mound revetments and breakwaters under random wave attack. The formulae are based upon a series of model tests. More than two hundred and fifty tests have been performed in order to ensure that all the relevant variables could be varied systematically. As a result of the present series of investigations the main shortcomings in Hudson-type formulae have been solved. Stability formulae are given which include the influence of wave period, number of waves, armour grading, spectrum shape, groupiness of waves and the permeability of the core. A clearly defined damage level parameter is introduced in the formulae.

INTRODUCTION

The use of coarse materials, such as gravel and natural stone for slope revetments and breakwaters, is very common in civil engineering. In recent years, there has been an increasing demand for reliable design formulae, because of the ever growing dimensions of the structures and the necessity to move into more hostile environments.

The Hudson formula is well known because of its simplicity. In the last decade, however, it has been found by many users to have a lot of shortcomings. It does not include, for example, the influence of the wave period and does not take into account random waves. The study of Ahrens (1975) in a large wave tank showed the importance of the wave period on the stability of riprap. The tests, however, were performed with regular waves. Evaluation of Ahrens' data by Pilarczyk and den Boer (1983) produced stability formulae which included the wave period. Losada and Giménez-Curto (1979) gave formulae for stability of rubble mound slopes under regular wave attack which also included the wave

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period. Hedar (1960) showed the importance of the permeability of the structure. His tests were also performed with regular waves.

An extensive investigation was performed by Thompson and Shuttler (1975) on the stability of rubble mound revetments under random waves. One of their main conclusions was, that within the scatter of the results, the erosion damage showed no clear dependence on the wave period. Re-analyzing their data, however, the author has found a very clear dependence on the wave period! Thompson and Shuttler, in fact, used steep waves with a small range of wave periods. Their work has been used as a starting point for the present research. The dependence of erosion damage on wave period has now been confirmed for a wider range of conditions by performing tests with longer wave periods. The dependency of erosion damage on other variables has also been considered in the present investigation.

The first part of the present investigation has been described by Van der Meer and Pilarczyk (1984). Stability formulae were given which were mainly based on rubble mound revetments with an impermeable core. Rubble mound breakwaters with a permeable core were subsequently tested later on and reported on by Van der Meer (1985). The last tests of the programme were finished early 1986. An analysis of the complete research programme is given here. This analysis has produced two design formulae which take into account all three hundred tests performed in the investigation and the results of Thompson and Shuttler (1975).

GOVERNING VARIABLES

A design formula for armour units should be a method of determining the minimum mass of individual armour units, for given mass densities, required for stability as a function of all the variables involved. In the following discussion the size of armour units is referred to as the average mass of graded rubble, W_{50}, or the nominal (cubical) diameter, D_{n50}, where:

$$D_{n50} = (W_{50}/\rho_a)^{1/3} \tag{1}$$

and where:

D_{n50}	=	nominal diameter	(m)
W_{50}	=	50% value of the mass distribution curve	(kg)
ρ_a	=	mass density of stone	(kg/m^3)

The relative mass density of the stone in water can be expressed by:

$$\Delta = \rho_a/\rho - 1 \tag{2}$$

where:

Δ = relative mass density (—)
ρ = mass density of water (kg/m^3)

As shown by many authors there are a large number of variables affecting armour stability. The variables investigated are shown in Table 1.

The Hudson formula can be rewritten in a very simple form by using the nominal diameter, D_{n50}, and the relative mass density, Δ, as:

$$H_s/\Delta D_{n50} = (K_D \cot \alpha)^{1/3} \tag{3}$$

The dimensionless wave height, $H_s/\Delta D_{n50}$, is the same as the frequently used stability number N_s.

The wave period can be related to external processes, for example waves breaking on a slope, by the dimensionless surf similarity parameter, ξ_z, where:

$$\xi_z = \tan \alpha / \sqrt{2\pi H_s / g T_z^2} \tag{4}$$

The average wave period, T_z, is used instead of the peak period, T_p. The use of T_z takes into account the influence of the spectrum shape on stability, where the peak period, T_p, does not. This aspect is treated later on in this paper. Plots of $H_s/\Delta D_{n50}$ (or N_s) versus ξ_z are used by many authors to show the influence of wave height, wave period and slope angle on armour stability.

The dimensionless damage level, S, can be described by:

$$S = A/D_{n50}^2 \tag{5}$$

where A is the eroded cross-sectional area of the profile, see Fig. 1. A physical description of S is the number of cubic stones with a side of D_{n50}, eroded within a width of one D_{n50}. The "no damage" criterion of Hudson and Ahrens is taken

TABLE 1

Primary variables affecting armour stability

Variable	Symbol	Dimension
nominal diameter	D_{n50}	m
relative mass density	Δ	—
significant wave height	H_s	m
average wave period	T_z	s
slope angle	α	degr.
damage level	S	—
number of waves	N	—
armour grading	D_{85}/D_{15}	—
spectrum shape	$\epsilon_{5\%}$, Q_p*	—
groupiness of waves	GF, $\overline{j_1}$, $\overline{j_2}$*	—
permeability of core	P	—
gravity	g	m/s^2

*Described in detail by Van der Meer and Pilarczyk (1984).

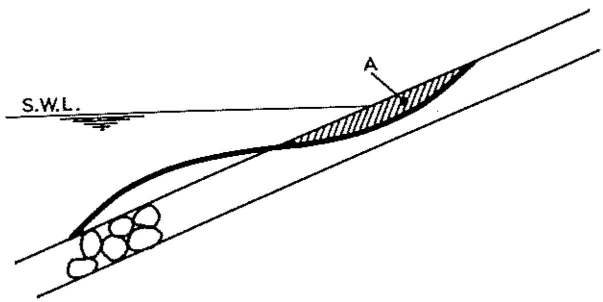


Fig. 1. Damage profile with erosion area A.

generally to be when S is between 1 and 3 and "failure" of the slope is assumed when S is greater than 10. The exact value of S for these criteria is dependent to some extent on the slope of the revetment. The advantage of this definition of damage is that it is independent on the length of the slope.

TEST EQUIPMENT, MATERIALS, PROCEDURE AND TEST PROGRAMME

All tests were conducted in a 1.0-m-wide, 1.2-m-deep and 50.0-m-long wave flume with the test section installed about 44 m from the random wave generator. A system developed by Delft Hydraulics was used to measure and compensate for reflected waves at the wave board. With this system standing waves and basin resonance were avoided. The incident significant wave height was measured with the structure in the flume, by means of two wave gauges, placed apart a quarter of the wave length.

For the investigation a surface profiler was developed with nine gauges placed 0.10 m apart on a computer-controlled carriage. The surface along the slope was measured every 0.040 m. Depending on the slope angle every survey consisted between 500 and 1600 data points. Successive soundings were taken at exactly the same points using the relocatability of the profiler. An average profile was calculated and plotted by computer and used for determining the erosion damage, S.

Broken stone was used for the armour layer, the main characteristics of which were: $W_{50}=0.123$ kg; $\rho_a=2630$ kg/m^3; $D_{n50}=0.036$ m; layer thickness 0.080 m. The sieve analysis curves were straight lines on a log-linear plot, see Fig. 2. Two gradings were used: $D_{85}/D_{15}=2.25$ (riprap) and 1.25 (uniform stones) respectively. The filter layer was defined by: D_{n50} (armour)/D_{n50} (filter) $=4.5$ and $D_{85}/D_{15}=2.25$, according to the tests of Thompson and Shuttler (1975). The thickness of the filter layer was 0.02 m. When an impermeable core was being tested the filter layer was placed directly on a slope constructed of mortar. When a permeable core was tested the armour layer was placed directly on the core, without a special filter layer. During the tests with a permeable core

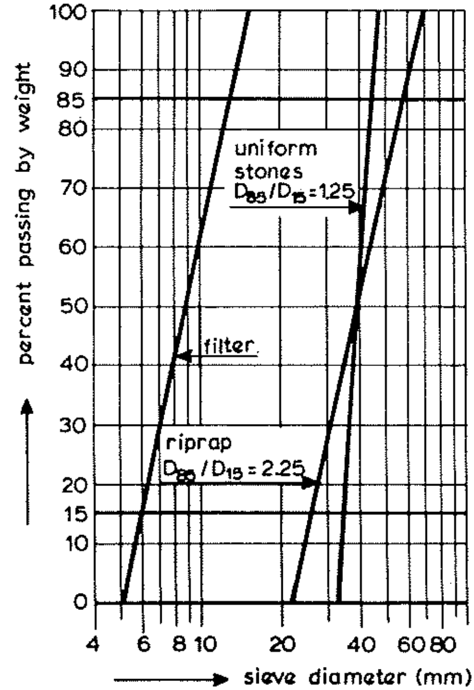


Fig. 2. Sieve curves.

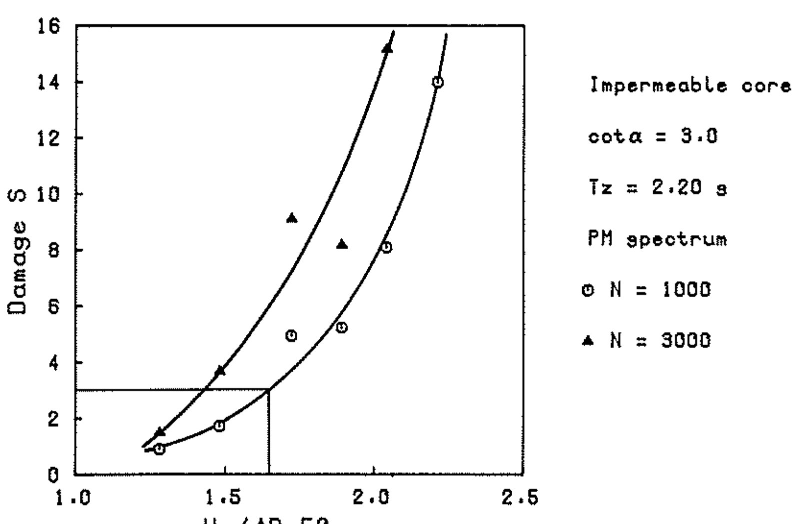


Fig. 3. Example of damage curves.

TABLE 2

Test programme

Slope angle cot α	Grading D_{85}/D_{15}	Spectrum shape	Core permeability	Relative mass density Δ	Number of tests
2	2.25	PM	none	1.63	19
3	2.25	PM	none	1.63	20
4	2.25	PM	none	1.63	21
6	2.25	PM	none	1.63	26
3	1.25	PM	none	1.62	21
4	1.25	PM	none	1.62	20
3	2.25	narrow	none	1.63	19
3	2.25	wide	none	1.63	20
3	1.25	PM	permeable	1.62	19
2	1.25	PM	permeable	1.62	20
1.5	1.25	PM	permeable	1.62	21
2	1.25	PM	homogeneous	1.62	16
2	1.25	PM	permeable	0.95	10
2	1.25	PM	permeable	2.05	10

PM = Pierson-Moskowitz spectrum.

the grading of the core was: $D_{85}/D_{15}=1.50$ with $D_{n50}=0.011$ m. This means that for the tests with a permeable core, $D_{n50}(\text{armour})/D_{n50}(\text{core})=3.2$.

Each complete test considered of a pre-test sounding, a test of 1000 waves, an intermediate sounding, a test of 2000 more waves, a final sounding. After each complete test the armour layer was removed and rebuilt. A test series consisted generally of five tests with the same wave period, but different significant wave heights. Wave heights ranged from 0.05 m to 0.26 m and wave periods from 1.3 to 3.2 s. A water depth of 0.80 m was applied for all tests. A damage curve was drawn for $N=1000$ and $N=3000$, for each test series, as shown in Fig. 3, where N is number of waves, in the test. From this curve the $H_s/\Delta D_{n50}$ value was taken for several damage levels, S, and the surf similarity parameter, ξ_z, given in eqn. (4) was calculated, using the average wave period, T_z, of the test series.

For example, the curve for $N=1000$ gives $H_s/\Delta D_{n50}=1.64$ for the damage level $S=3$, see Fig. 3. With $\Delta=1.615$, $D_{n50}=0.036$ m, $T_z=2.20$ s and cot $\alpha=3$, the surf similarity parameter becomes $\xi_z=2.97$, eqn. (4). The calculated values were used in $H_s/\Delta D_{n50}-\xi_z$ plots. The damage levels were chosen between start of damage and failure of the slope (filter layer visible).

The test programme is summarized in Table 2.

MODEL TEST RESULTS

Most of the model test results have already been published in the form of $H_s/\Delta D_{n50}-\xi_z$ plots by Van der Meer and Pilarczyk (1984) and Van der Meer

(1985). Due to space limitations these plots are not repeated here. All test results, however, are given in Figs. 9 and 10.

Influence of armour layer grading

Tests were carried out at slopes with cot $\alpha=3.0$ and cot $\alpha=4.0$ with widely graded riprap, $D_{85}/D_{15}=2.25$, and uniform stones, $D_{85}/D_{15}=1.25$. Test results are described by Van der Meer and Pilarczyk (1984). The damage to both gradings was found to be the same. It can be concluded that the grading of the armour within the range tested has no influence on the stability and that, within this range, the armour layer can be described simply by the nominal diameter, D_{n50}.

Influence of spectrum shape and groupiness of waves

The main part of the present series of tests was conducted with a Pierson Moskowitz (PM) spectrum. The test series with a slope angle of cot $\alpha=3.0$ was performed with a very narrow spectrum and also a wide spectrum. A comparison of these spectra (with the same H_s and T_z) is shown in Fig. 4. The test results have been described in detail by Van der Meer and Pilarczyk (1984) and are shown in Fig. 5.

Using the average wave period, T_z, for calculating the surf similarity parameter, ξ_z – eqn. (4), gave good agreement between the stability test results for a narrow and a wide spectrum. Using the peak period, T_p, the results of the wide spectrum in Fig. 5 will shift to the right and large differences will exist between the narrow and wide spectrum. It can be stated, therefore, that stability is not influenced, or only slightly influenced by the spectrum shape or by the groupiness of waves, using the average period, T_z. This conclusion was also reached for the development of gravel beach profiles, by Van Hijum and Pilarczyk (1982).

Influence of number of waves

In the investigation of Thompson and Shuttler (1975) profiles were sounded after every 1000 waves, up to $N=5000$. The storm duration showed a large influence on stability, using random waves. Stability is often reached within a short period of time when monochromatic waves are used. Therefore, a large difference exists between testing with monochromatic and random waves.

Analyzing the results of Thompson and Shuttler gives the relationship between damage and number of waves shown in Fig. 6. Data points are generally based on 50 tests and are independent of slope angle, wave period and damage level. Since a clear relationship was found between damage and number of waves, it was decided that the total number of waves in the present study

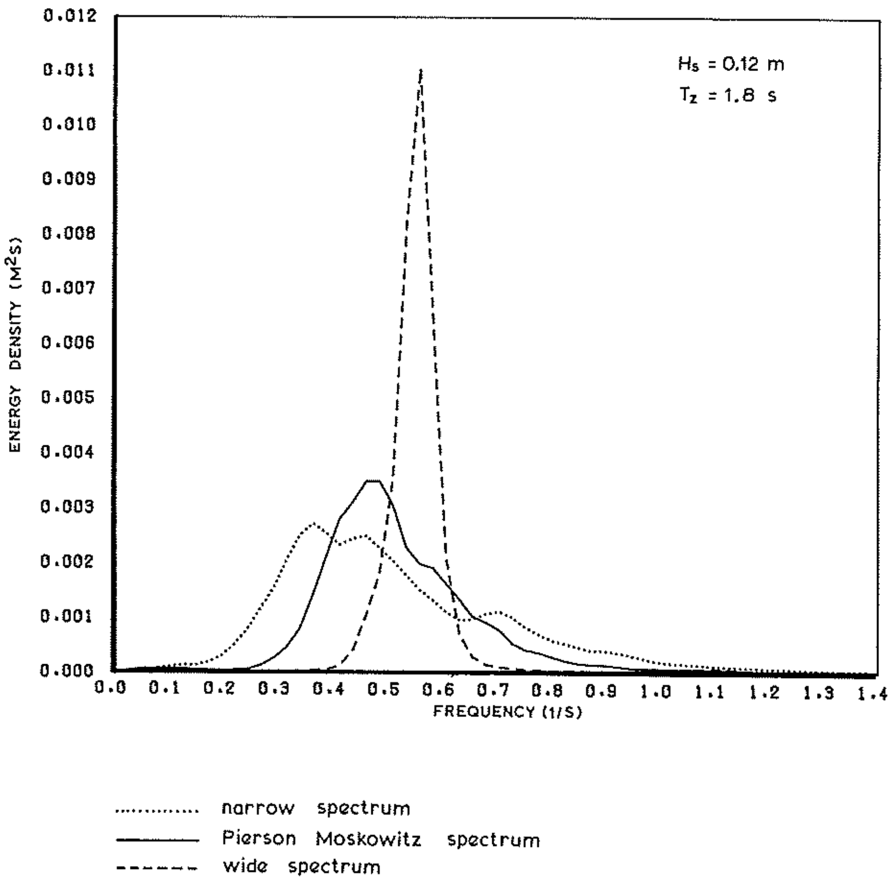


Fig. 4. Spectra with the same H_s and T_z.

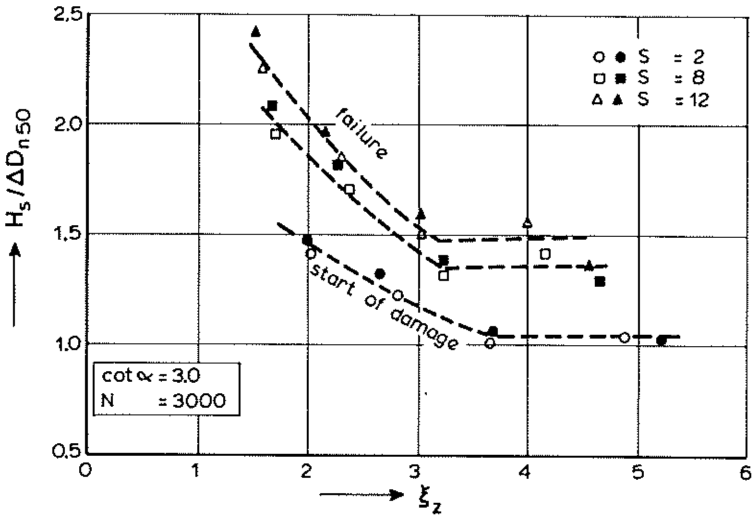
could be reduced to $N=3000$ with one intermediate sounding after $N=1000$. Analyzing the ratio of damage after 3000 and 1000 waves and using the damage after 3000 waves as a reference, gave a new point (*) in Fig. 6. The difference between the results of the two investigations is small.

The relationship between damage, S, and number of waves, N, can be described by a square root function when N is between 1000 and 7000 waves, as shown in Fig. 6. For this range the influence of the number of waves on stability can be described simply by the parameter group $S/\sqrt{N}$.

Thompson and Shuttler performed five long duration tests with N upto 15,000. These data are also given in Fig. 6 together with the data of 14 tests with $N=500$.

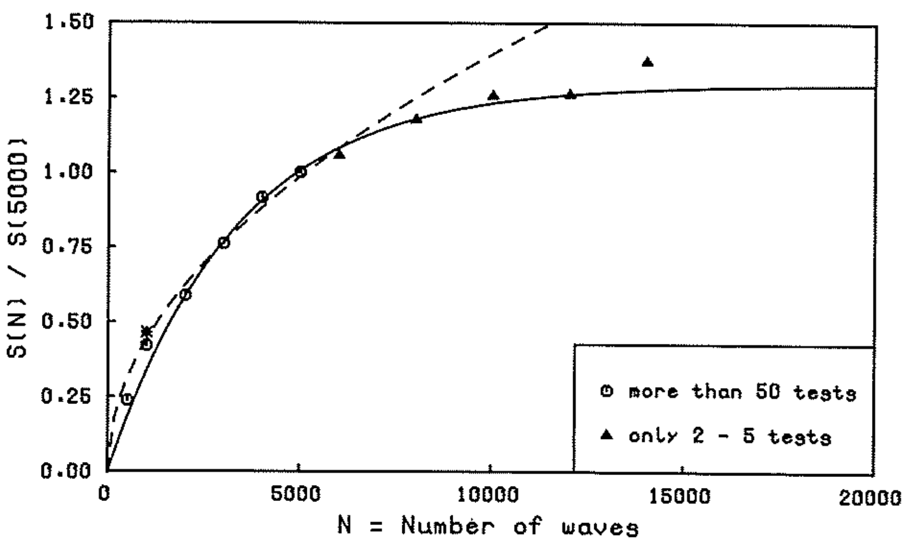
A function which describes the influence of the storm duration on stability completely should meet the following requirements:

— From $N=0$ to $N=500$ or 1000 the function should be almost linear



open symbols : wide spectrum, $\varepsilon_{5\%}$ = 0.59, GF = 0.72
closed symbols : narrow spectrum, $\varepsilon_{5\%}$ = 0.10, GF = 0.99

Fig. 5. Influence of spectrum shape on stability.



S(N)/S(5000) = 1.3(1-exp(-0.0003N))

S(N)/S(5000) = 0.014√N

Fig. 6. Influence of number of waves on damage.

TABLE 3

Lower and upper damage levels

Slope angle cot α	Damage level, S	
	Onset of damage	Failure
1.5	2	8
2	2	8
3	2	12
4	3	17
6	3	17

— For large N numbers there should be reached a limit to the damage (equilibrium).

A function which meets this requirements is:

$$f(S)=a[1-\exp(-bN)] \quad (6)$$

where a and b are curve-fitting coefficients. Based on the data of Fig. 6 the coefficients, a and b, are found to be 1.3 and 3×10^{-4} respectively. The influence of the storm duration on stability for the whole range of N can therefore be described by:

$$S(N)/S(5000)=1.3\ [1-\exp(-3\times10^{-4}\ N)] \quad (7)$$

The damage is limited to 1.3 times the damage found after $N=5000$. This factor is also found using $N=8500$ in the $S/\sqrt{N}$ relation. The parameter group $S/\sqrt{N}$ covers a large part of the area of interest and will, for sake of simplicity be included in the stability formula to be developed. For $N<1000$ and $N>7000$ one can use eqn. (7).

Influence of wave height, wave period and slope angle

The extent of damage depends on the slope angle. More stones have to be displaced for gentler slopes before the failure criterion, (filter layer visible), is reached, as a larger area is present around the water level. The lower and upper damage levels, that is the onset of damage and failure, were determined from the investigations and are shown in Table 3.

The influence of the wave height, wave period and slope angle is shown in Fig. 7. The dimensionless wave height, $H_s/\Delta D_{n50}$, is plotted against the surf similarity parameter, ξ_z. Results are shown for a structure with an impermeable core and after a wave attack of 3000 waves. The damage level, $S=5$, is plotted for slope angles with cot $\alpha=2$, 3, 4 and 6 respectively. The curves are the stability formulae which are derived later in the paper.

Figure 7 shows the same trend as that found by Ahrens (1975) for regular

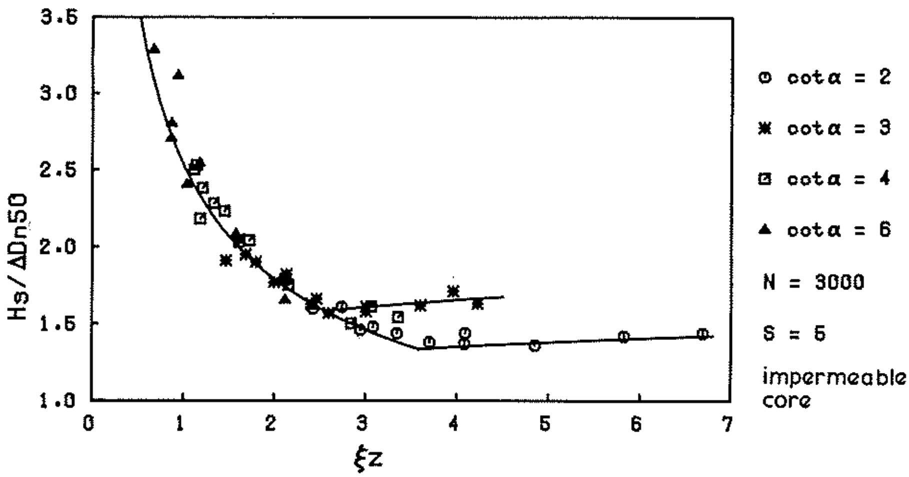


Fig. 7. Influence of wave height, period and slope angle on stability.

waves. Plunging waves are present on the left side of the figure for $\xi_z < 3$. Surging waves are shown when $\xi_z > 3$. Minimum stability is found for the transition from plunging to surging waves, referred to as collapsing waves. For plunging (*breaking*) waves the surf similarity parameter, ξ_z, or *breaker* parameter gives a good description of the influence of slope angle and wave steepness on stability. For surging waves on the right side of Fig. 7, different curves are shown for different slope angles. The transition from plunging to surging waves shifts to the right (larger ξ_z-value) for steeper slopes.

Influence of permeability

Three structures have been tested: A revetment with an impermeable core, clay or sand in nature and concrete in model, tested first. A structure with a permeable core (breakwater) tested second. This structure was much more stable than the impermeable revetment. A homogeneous structure consisting only of armour stones which is an upper boundary, as far as permeability is concerned, tested third. The impermeable revetment can be regarded as a lower boundary of permeability. The coefficient P was introduced to take account of permeability, described later in the paper.

The results for these three structures are shown in Fig. 8. Results are shown for a slope with cot $\alpha = 2$, a damage level of $S = 5$, and after 3000 waves. The influence of the wave period for plunging waves (left-hand side of figure) shows the same trend for all three structures, although a more permeable structure is more stable. A more permeable structure is also more stable for surging waves ($\xi_z > 3.5$), but the stability increases with larger wave periods. The curves are steeper for larger permeability.

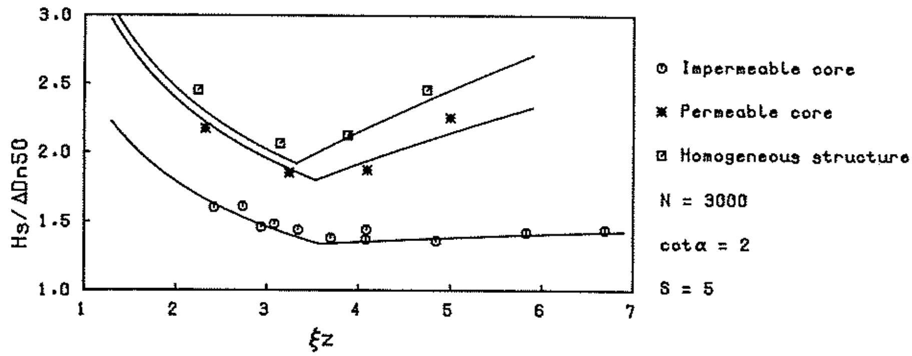


Fig. 8. Influence of permeability on stability.

This phenomenon can be explained in physical terms by the difference in water motion on the slope. For a slope with an impermeable core the flow is concentrated in the armour layer causing large forces on the stones during rundown. For a slope with a permeable core the water dissipates into the core and the flow becomes less violent. With longer wave periods (larger ξ_z) more water can percolate and flow down through the core. This reduces the forces and stabilizes the slope.

Influence of relative mass density

Tests were performed with stones having different mass densities. The light stones (broken bricks) had a mass density of 1950 kg/m^3 and the heavy stones (basalt) of 3050 kg/m^3. Normal stones had a mass density of 2620 kg/m^3. Results for light, normal and heavy stones are shown in Fig. 9 for the onset of damage, $S=2$. Both the light and heavy stones are a little more stable than the

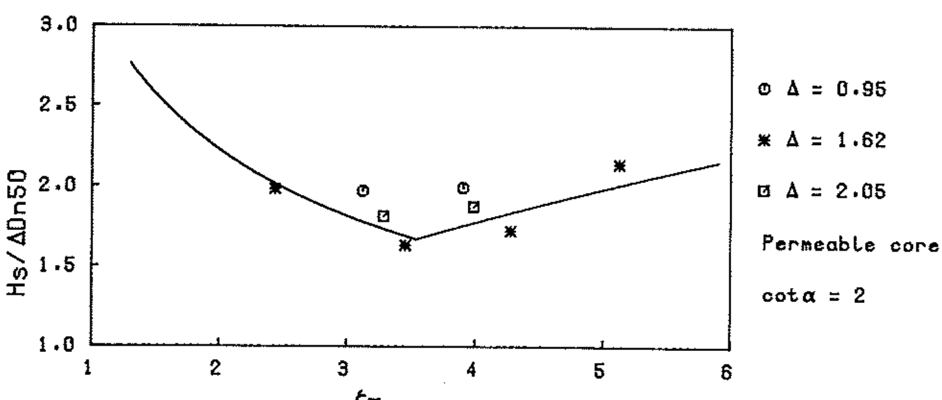


Fig. 9. Influence of relative mass density on stability.

normal stones. Within the scatter of the results it can be concluded that the influence of the mass density can be described by $H_s/\Delta D_{n50}$.

DERIVATION OF STABILITY FORMULAE

The first tests of the investigation were performed in 1983. The last test was finished in March 1986. The results were analyzed initially on the basis of the number of tests performed, the test program being changed in accordance with the results of this initial analysis. The initial analyses were reported by Van der Meer and Pilarczyk (1984) and Van der Meer (1985). All results have been re-analyzed for the present paper, keeping in mind the initial results and taking into account the tests performed after publication of Van der Meer (1985). The formulae derived in the present report can be regarded as the definitive result of the investigation.

The list of variables given in Table 1 can be shortened using the results described above. The influence of the number of waves is given by $S/\sqrt{N}$. The influence of the relative mass density is given correctly by $H_s/\Delta D_{n50}$. Armour grading, spectrum shape and wave groupiness have practically no influence on the stability (using T_z) and can, therefore, be deleted. The stability of rubble mound revetments and breakwaters can then be described by the following dimensionless variables:

$H_s/\Delta D_{n50}$; ξ_z; cot α; $S/\sqrt{N}$; permeability P

All results show a clear difference between plunging and surging waves. Therefore two stability formulae have been considered, one for plunging waves and one for surging waves.

Plunging waves

The influence of wave period (or steepness) and slope angle can be described by the breaker parameter ξ_z, see Figs. 7–9. These influences can be described by a power function:

$$H_s/\Delta D_{n50} = a\,\xi_z^b \tag{8}$$

where a and b are curve-fitting coefficients. The coefficient b was determined by regression analyses on the actual test results and was equal to -0.54, -0.51 and -0.51 for the impermeable core, permeable core and homogeneous structure respectively. The factor b showed no dependency on damage level, slope angle or storm duration. A value of -0.5 was chosen for b.

The damage level and number of waves can be described by $S/\sqrt{N}$. This influence of the damage level and the number of waves can also be described by a power function:

$$H_s/\Delta D_{n50} \times \sqrt{\xi_z} = a(S/\sqrt{N})^b \tag{9}$$

In this case the value of the coefficient, b, was 0.22, 0.17 and 0.19 for the impermeable core, permeable core and homogeneous structure, respectively. An average value of 0.2 was chosen. This changes eqn. (9) into:

$$H_s/\Delta D_{n50} \times \sqrt{\xi_z} = a(S/\sqrt{N})^{0.2} \tag{10}$$

The coefficient, a, is only dependent on the permeability of the structure. The following formulae were derived for the three structures tested:

$$\text{impermeable core: } H_s/\Delta D_{n50} \times \sqrt{\xi_z} = 4.1\ (S/\sqrt{N})^{0.2} \tag{11}$$

$$\text{permeable core: } H_s/\Delta D_{n50} \times \sqrt{\xi_z} = 5.3\ (S/\sqrt{N})^{0.2} \tag{12}$$

$$\text{homogeneous structure: } H_s/\Delta D_{n50} \times \sqrt{\xi_z} = 5.7\ (S/\sqrt{N})^{0.2} \tag{13}$$

Formulae (11) to (13) can be combined into one formula when the permeability coefficient, P, is introduced.

Surging waves

The same kind of procedure can be followed for surging waves, although the breaker parameter does not cover the influence of the slope angle. The influence of the wave steepness is described by:

$$H_s/\Delta D_{n50} = a\ \xi_z^{\ b} \tag{8}$$

For the three structures tested, impermeable core, permeable core and homogeneous structure, the value of the coefficient b, was 0.1, 0.5 and 0.6, respectively. The increasing value of b shows the increasing influence of wave steepness with increasing permeability. This reflected in the steeper curves found on the $H_s/\Delta D_{n50} - \xi_z$ plot, see Fig. 8. The influence of the slope angle can be described by:

$$\text{impermeable core: } H_s/\Delta D_{n50} = a \cot \alpha^b \xi_z^{\ 0.1} \tag{14}$$

$$\text{permeable core: } H_s/\Delta D_{n50} = a \cot \alpha^b \xi_z^{\ 0.5} \tag{15}$$

A slope with $\cot \alpha = 2$ was tested for the homogeneous structure; the influence of slope angle was assumed to be the same for impermeable and permeable core tests. The coefficient, b, in eqns. (14) and (15) was found to be 0.46 and 0.54, respectively. A value of 0.5 was selected for the present study which resulted in the parameter $\sqrt{\cot \alpha}$. The influence of damage level and number of waves can then be described by:

$$\text{impermeable core: } H_s/\Delta D_{n50} = a(S/\sqrt{N})^b \sqrt{\cot\alpha}\, \xi_z^{0.1} \tag{16}$$

$$\text{permeable core: } H_s/\Delta D_{n50} = a(S/\sqrt{N})^b \sqrt{\cot\alpha}\, \xi_z^{0.5} \tag{17}$$

$$\text{homogeneous structure: } H_s/\Delta D_{n50} = a(S/\sqrt{N})^b \sqrt{\cot\alpha}\, \xi_z^{0.6} \tag{18}$$

The coefficient, b, in eqns. (16)–(18) was found to be 0.17, 0.19 and 0.25, respectively. A value of 0.2 was selected for the present study. The coefficient, a, in eqns. (16)–(18) is only dependent on the permeability of the structure. The following formulae were derived for the three structures tested, by curve-fitting of the coefficient a in eqns. (16)–(18) and using $b = 0.2$:

$$\text{impermeable core: } H_s/\Delta D_{n50} = 1.35\ (S/\sqrt{N})^{0.2} \sqrt{\cot\alpha}\, \xi_z^{0.1} \tag{19}$$

$$\text{permeable core: } H_s/\Delta D_{n50} = 1.07\ (S/\sqrt{N})^{0.2} \sqrt{\cot\alpha}\, \xi_z^{0.5} \tag{20}$$

$$\text{homogeneous structure: } H_s/\Delta D_{n50} = 1.10\ (S/\sqrt{N})^{0.2} \sqrt{\cot\alpha}\, \xi_z^{0.6} \tag{21}$$

The permeability coefficient, P

A permeability coefficient, P, was introduced into the stability formulae to take into account the permeability of the structure, see eqns. (22) and (23). This permeability coefficient has no physical meaning, but was introduced to ensure that permeability is taken into account. In eqns. (19)–(21) the power coefficient of ξ_z has a value, dependent on the permeability of the structure, of 0.1, 0.5 and 0.6 respectively. Therefore, P is defined by $P = 0.1$ for the impermeable core, 0.5 for the permeable core, and 0.6 for the homogeneous structure. Now the coefficients 0.1, 0.5 and 0.6 in eqns. (19)–(21) can be replaced by this P.

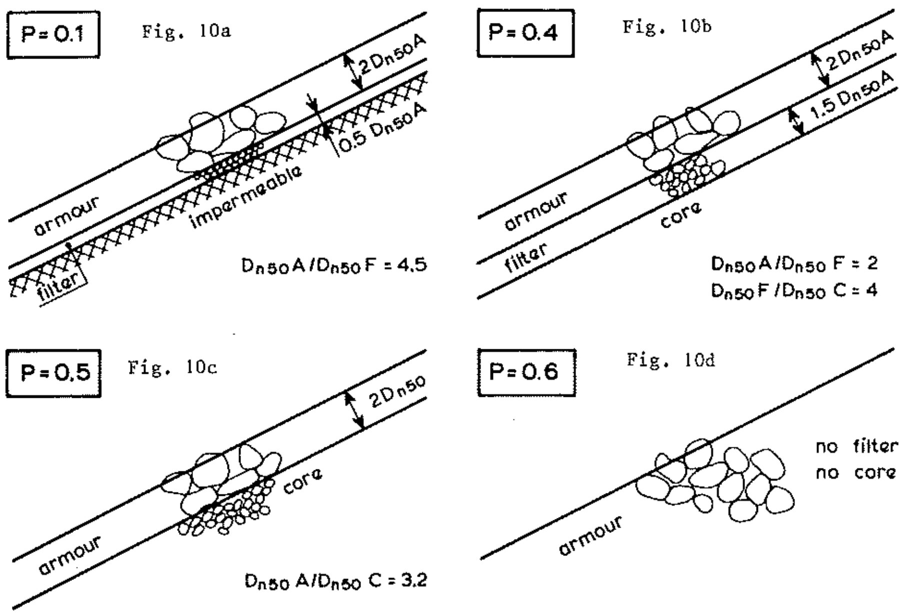
Four structures are shown in Fig. 10 for which three P values, 0.1, 0.5 and 0.6. The other structure has an assumed value of 0.4. In the absence of other information the selection of the P-value is left to the engineers judgement. Further research will probably provide a more physical definition of P, P can then be calculated for each particular structure.

The coefficients a, 4.1, 5.3 and 5.7 in eqns. (11)–(13) and 1.35, 1.07 and 1.10 in eqns. (19)–(21) can now be described as a function of the permeability coefficient, P. It must be remembered that the impermeable structure and the homogeneous structure are in fact lower and upper boundaries of permeability. The permeable core is a structure lying between the boundaries. The permeability coefficient can be included in the formulae as follows:

$$H_s/\Delta D_{n50} \times \sqrt{\xi_z} = a\, P^b (S/\sqrt{N})^{0.2} \qquad \text{for plunging waves} \tag{22}$$

and

$$H_s/\Delta D_{n50} = a\, P^b (S/\sqrt{N})^{0.2} \sqrt{\cot\alpha}\, \xi_z^{P} \qquad \text{for surging waves} \tag{23}$$



$D_{n50}A$ = nominal diameter of armour stone

$D_{n50}F$ = nominal diameter of filter material

$D_{n50}C$ = nominal diameter of core

The structures on Fig.10a, 10b and 10c have been tested

The value of P for Fig.10d has been assumed

Fig. 10. Permeability coefficient assumptions for various structures.

Curve fitting of a P^b in eqns. (22) and (23) to the established P-values and the coefficients in eqns. (11)–(13) and (19)–(21) gives the final formulae. These formulae are:

$$\bigstar\ H_s/\Delta D_{n50} \times \sqrt{\xi_z} = 6.2\, P^{0.18} (S/\sqrt{N})^{0.2} \qquad \text{for plunging waves} \qquad (24)$$

and

$$\bigstar\ H_s/\Delta D_{n50} = 1.0\, P^{-0.13} (S/\sqrt{N})^{0.2} \sqrt{\cot \alpha}\, \xi_z^{P} \qquad \text{for surging waves} \qquad (25)$$

The coefficient -0.13 in formula (25) suggests that the stability will decrease with increasing permeability. This is in contrast to the results found in model tests. The influence of the permeability, in the surging waves region is, however, described by the factor $P^{-0.13}\xi_z^{P}$, a factor which increases in this region with increasing P.

Formulae (24) and (25) are shown in Figs. 7 and 8. The transition from

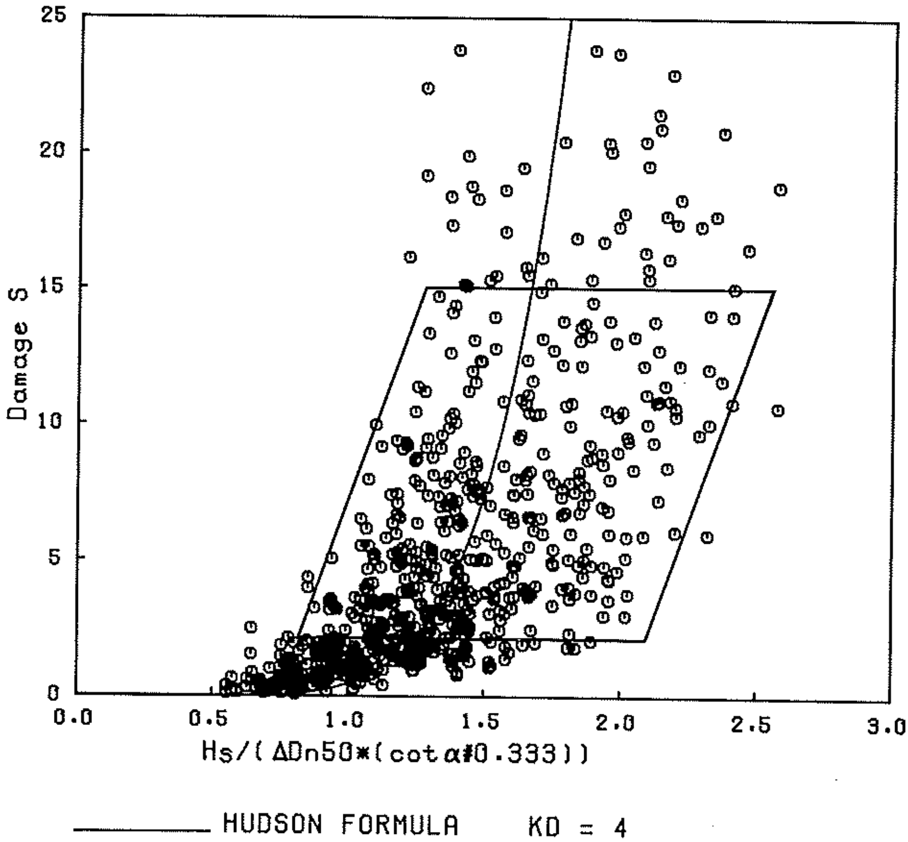


Fig. 11. Comparison of the Hudson Formula with the test results.

plunging waves, formula (24), to surging waves, formula (25), can be calculated using:

$$\xi_z = (6.2\, P^{0.31} \sqrt{\tan\alpha})^{1/(P+0.5)} \tag{26}$$

Depending on slope angle and permeability the transition lies between $\xi_z = 2.5$ and 4.

Comparison the design formulae proposed with test results

The Hudson formula, eqn. (3), only considers the "no-damage" criterion. The Shore Protection Manual (1984), however, gives the influence of the damage level on the K_D factor. Using the data for rough quarry stone the Hudson formula can be transformed into an expression which takes into account the damage level:

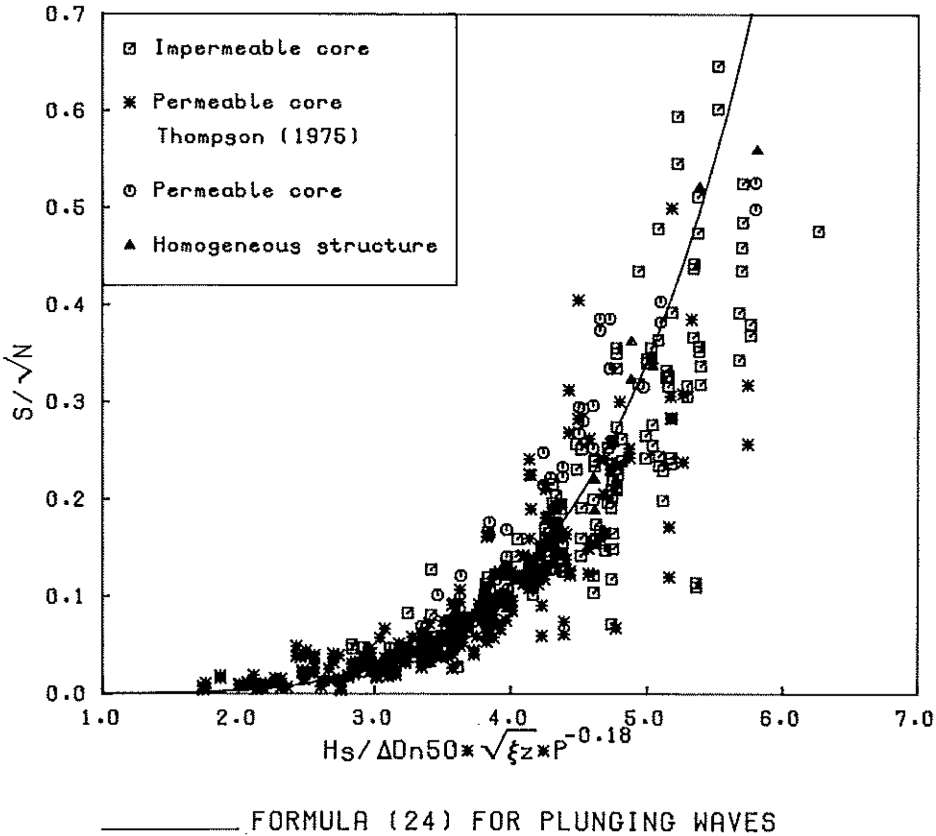


Fig. 12. Stability Formula (24) for plunging waves with actual test results.

$$H_s/\Delta D_{n50} = 1.11\ (\cot\alpha)^{1/3} S^{0.15} \tag{27}$$

(for $K_D = 4$, $S = 2$ equivalent to 2.5% damage and $H_{10} = 1.27H_s$).
Formula (27) can directly be compared with the model test results obtained in the present investigation. Figure 11 shows this comparison with the area of interest for the designer (between onset of damage and failure). It is clear that although the Hudson formula is, in fact, a reasonable average of the test results, it can only be used to give a rough estimate for a particular case.

Formulae (24) and (25) are shown on Figs. 12 and 13 together with all the actual test results. On both figures the vertical axis is the parameter $S/\sqrt{N}$. The data for an impermeable core, a permeable core, a homogeneous structure, and for the data of Thompson and Shuttler (1975) is distinguished in these figures. In total more than 650 data points have been plotted in the two figures.

The stability formulae (24) and (25) agree well with the test results and are a substantial improvement in comparison with the Hudson formula.

Figures 12 and 13 also provide a basis for describing the accuracy of For-

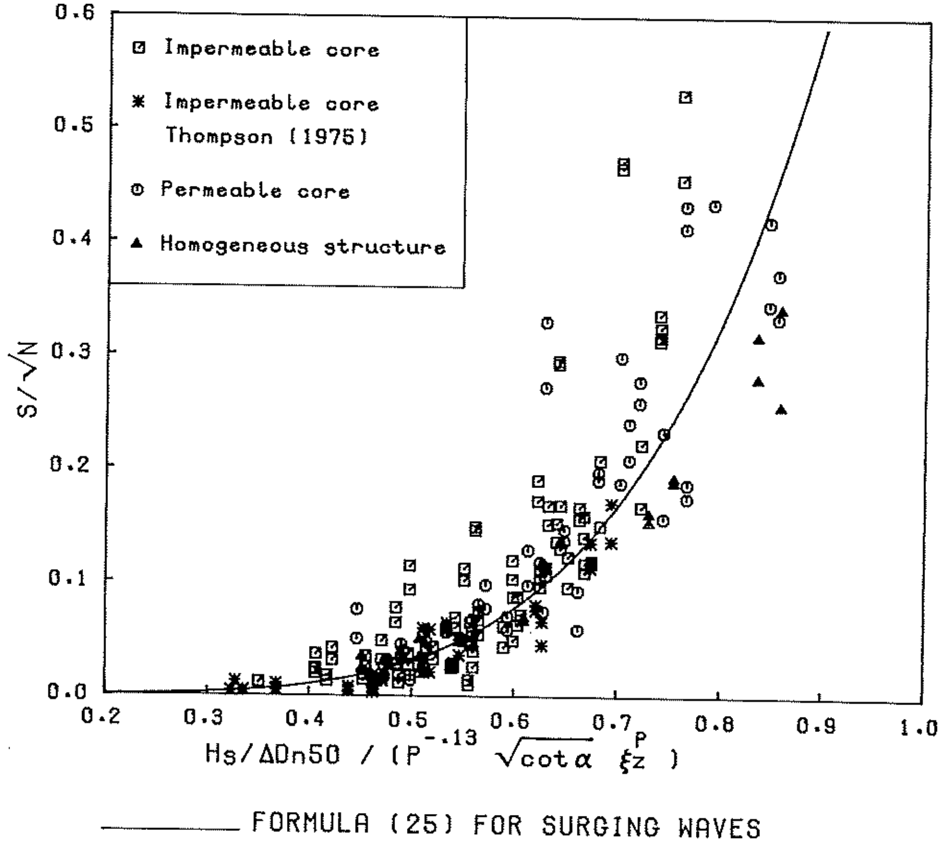


Fig. 13. Stability formula (25) for surging waves with actual test results.

mulae (24) and (25). The scatter in results is due to the scatter likely to occur in nature and the scatter due to curve fitting. Within 500 datapoints for 15 cases it was found that a lower wave height gave more damage than a 10–20% higher wave height, the other factors being the same. In fact, Fig. 3 shows one of these cases. Two tests in 250 gave a damage factor that was 70% higher than expected. These tests were repeated and then gave the expected damage. This scatter will also be present in nature.

The coefficients 6.2 in formula (24) and 1.0 in formula (25) give the average curves through the test results. Assuming a normal distribution the standard deviation of these coefficients can be calculated. This results in a standard deviation of 0.4 (6.5%) for the coefficient 6.2 and 0.08 (8%) for the coefficient 1.0. These standard deviations take into account the scatter likely to occur in nature and the scatter due to curve fitting and can be used in probabilistic design procedures or to establish confidence levels.

Formulae (24) and (25) can also be rewritten as reliability functions for use in probabilistic calculations. This procedure is described by Van der Meer and Pilarczyk (1987).

All results were based on small-scale tests. It was stated before the start of the investigation, however, that large-scale tests should be performed in order to evaluate scale effects on stability. Eleven tests were repeated in the Delft Hydraulics large wave flume. Tests on a slope with cot $\alpha=3$ and with both a permeable and impermeable core were performed. The linear factor between the small- and large-scale tests amounted to 6.25. The armour layer consisted of 15–55 kg stones. This investigation will be described elsewhere. The conclusion, however, was very clear: The large-scale tests showed no influence of scale effects on the stability of the armour layer of rock, in comparison with the small-scale tests.

Formulae (24) and (25) are based on physical model tests. Encouraging results have also been obtained by Kobayashi and Jacobs (1985) using a mathematical model for the flow characteristics in the downrush to predict the stability of riprap. There is quantitative agreement between these results and the results of Ahrens (1975). It is very difficult, however, to include different damage levels and storm durations in such models and it will be some time before a complete mathematical model is available to describe stability of rubble mounds.

CONCLUSIONS

(1) Practical design formulae have been developed for rubble mound revetments and breakwaters under random wave attack based on more than three hundred model tests and on the work of Thompson and Shuttler (1975).

(2) The stability has been determined in a dimensionless form, using:
— the dimensionless wave height parameter: $H_s/\Delta D_{n50}$
— the breaker parameter: ξ_z
— the slope angle: cot α
— the damage as function of number of waves: $S/\sqrt{N}$
— the permeability of the core: P.

(3) Within the conditions tested the following parameters did not influence the stability:
— the grading of the armour
— the spectrum shape
— the groupiness of waves.

(4) The scatter in the results which is likely to occur in nature and that due to curve fitting can be covered by standard deviations calculated for each formula. These standard deviations can be used in probabilistic design.

(5) Large-scale tests confirmed the validity of the small-scale tests. No scale

effects were established on the stability of the armour layer, using a linear scale factor of 6.25.

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