DETERMINISTIC AND PROBABILISTIC DESIGN OF BREAKWATER ARMOR LAYERS

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ABSTRACT: New stability formulas for rubble mound revetments and breakwaters under random wave attack have been established as a result of a comprehensive model investigation at Delft Hydraulics. These formulas include parameters such as the wave period, storm duration, and permeability of the structure and a clearly defined damage level. Main shortcomings in Hudson-type formulas have been solved. The development of these new formulas was described earlier; the present paper does not repeat this development. The stability formulas are given with the range of possible application of the parameters. It is shown how the formulas can be used in both deterministic and probabilistic design. The deterministic design procedure gives curves showing the influences of the various parameters. Probabilistic design results in curves showing the probability of exceedance of damage levels in the lifetime of the structure.

INTRODUCTION

The use of coarse materials, e.g., gravel and natural stone, for slope revetments and breakwaters is very common in civil engineering. In recent years, there has been an increasing demand for reliable design formulas because of the ever-growing dimensions of structures and the necessity to move into more hostile environments.

The Hudson formula has been found by many users to have a number of shortcomings. It does not include, for example, the influence of the wave period and does not take into account random waves. The study of Ahrens (1975) in a large wave tank showed the importance of the wave period on the stability of riprap. The tests, however, were performed with regular waves. Evaluation of Ahrens' data by Pilarczyk and den Boer (1983) produced stability formulas that included the wave period. Losada and Giménez-Curto (1979) gave formulas for the stability of rubble mound slopes under regular wave attack that also included the wave period. Hedar (1960, 1986) showed the importance of the permeability of the structure. His tests were also performed with regular waves.

An extensive investigation was performed by Thompson and Shuttler (1975) on the stability of rubble mount revetments under random waves. One of their main conclusions was that, within the scatter of the results, the erosion damage showed no clear dependence on the wave period. By reanalyzing their data, however, the writer found, in fact that there was a clear dependence on the wave period. The work of Thompson and Shuttler has been used, therefore, as a starting point for an extensive model research program. More than 250 physical model tests have been per-

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formed in addition to the more than 100 tests performed by Thompson and Shuttler. Analysis of the results from all of these tests has resulted in two practical design formulas that describe the influence of wave period, storm duration, armor grading, spectrum shape, grouping of waves, and the permeability of the core.

The development of these formulas has been described by Van der Meer (1987). The present paper restates the formulas and describes the relevant parameters. The formulas are then used in a deterministic approach to produce design graphs, showing the influence of the parameters. Finally the formulas have been rewritten in the form of reliability functions and have been used in a probabilistic design.

Comparison with Hudson Formula

The Hudson formula is well-known because of its simplicity. Stone mass, relative mass density, and wave height are directly related to the slope angle, given by cotα. Figs. 1 and 2 show this relationship. The

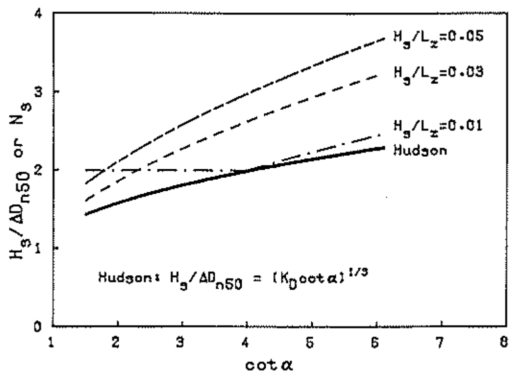


FIG. 1. Comparison Hudson and New Formulas for Permeable Core and after 1,000 Waves

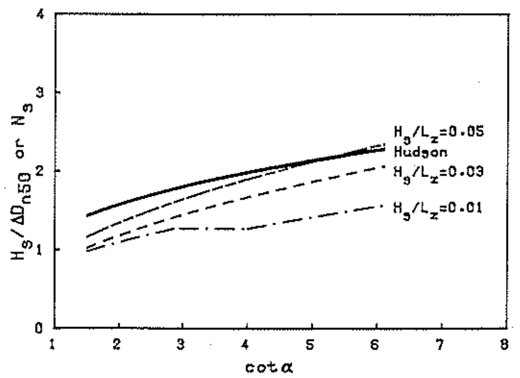


FIG. 2. Comparison Hudson and New Formulas for Impermeable Core and after 5,000 Waves

relationship is independent of wave period, storm duration, and permeability of the structure. Figs. 1 and 2 show the Hudson formula [$K_D = 4.0$ and $H_{10} = 1.27\ H_s$ in the *Shore Protection Manual* (SPM) (1984)] and are compared with the new stability formulas, described in the following. Curves have been drawn for several values of the fictatious wave steepness. The wave steepness is called fictatious as the wave height is taken at the toe of the structure and the wave length is calculated in deep water ($H_s/L_E = 2\pi H_s/gT_z^2$). Fig. 1 shows the curves for a permeable structure after a storm duration of 1,000 waves (a little more than the number used by Hudson). Fig. 2 shows the stability of an impermeable revetment after a wave attack of 5,000 waves (equivalent to 5–10 hr in nature).

The conclusion is clear. The Hudson formula can only be used as a very rough estimate for a particular case. It should be borne in mind that a difference of a factor of two in the N_s-number means a difference in the stone mass of a factor of eight. These figures should be sufficiently convincing for the designer to apply the new formulas instead of the simple Hudson formula.

Stability Formulas

The initial analysis of the results of the investigation and the development of the formulas are described in Van der Meer and Pilarczyk (1984) and Van der Meer (1985). The analysis of all the results of the investigation, which led to the final stability formulas, is given in Van der Meer (1987). Two stability formulas were derived, one for plunging (breaking) waves and one for surging (nonbreaking) waves (Fig. 3).

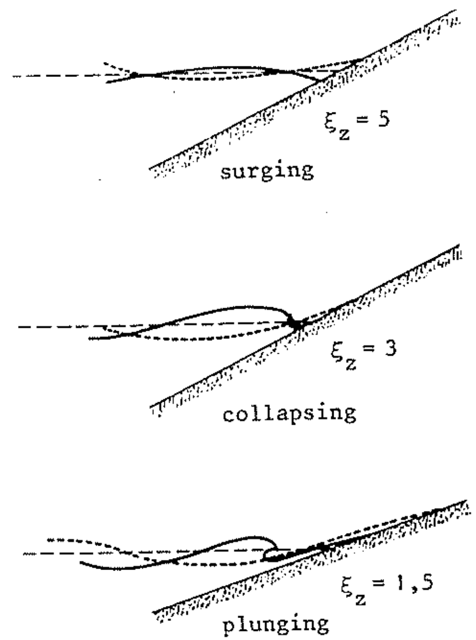


FIG. 3. Plunging, Collapsing, and Surging Waves

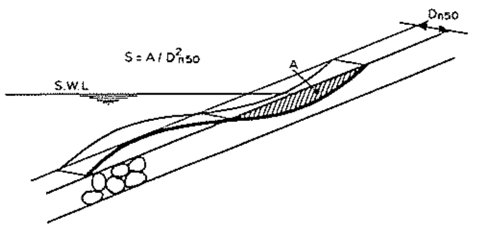


FIG. 4. Erosion Area and Damage Level S

The main basic assumptions for the formulas are: (1) A rubble mound structure with an armor layer consisting of rock; (2) there should be little or no overtopping (less than 10–15% of the waves); and (3) the slope of the structure should be generally uniform.

The formulas, which are described in the following sections, are:
For plunging waves:

$$\frac{H_s}{\Delta D_{n50}} * \sqrt{\xi_z} = 6.2P^{0.18}\left(\frac{S}{\sqrt{N}}\right)^{0.2} \quad \text{.......... (1)}$$

and for surging waves:

$$\frac{H_s}{\Delta D_{n50}} = 1.0P^{-0.13}\left(\frac{S}{\sqrt{N}}\right)^{0.2} \sqrt{\cot\alpha}\ \xi_z^P \quad \text{.......... (2)}$$

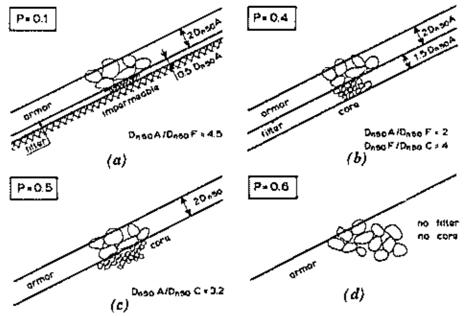
in which H_s = the significant wave height at the toe of the structure; ξ_z = the surf similarity parameter ($\xi_z = \tan\alpha/\sqrt{2\pi H_s/gT_z^2}$); T_z = the average wave period; α = the slope angle; Δ = the relative mass density of the stone ($\Delta = \rho_a/\rho - 1$); ρ_a = the mass density of the stone; ρ = the mass density of water; D_{n50} = the nominal diameter of the stone [$D_{n50} = (W_{50}/\rho_a)^{1/3}$]; W_{50} = the 50% value of the mass distribution curve; P = the permeability coefficient of the structure; S = the damage level ($S = A/D_{n50}^2$; A = the erosion area in a cross section (Fig. 4); and N = the number of waves (storm duration).

Governing Variables

1. The slope—The slope, cotα, should lie between 1.5 and 6.

2. The wave height—The significant wave height is used. In the tests performed in order to establish stability formulas, the significant wave height was defined as the average of the highest 1/3 of the waves. This definition is about equal to the significant wave height derived from the spectrum: $H_s = 4\sqrt{m_0}$, where m_0 = the zeroth moment of the energy density spectrum. Both can be used in the formulas.

A small number of tests were performed with a 1:30 uniform foreshore in front of the structure, causing breaking waves on the foreshore. Analysis of these tests indicated that if the structure is located in relatively shallow water and that if the wave height distribution is truncated, the 2% value of the wave height exceedance curve gives the best agreement with results showing a Rayleigh distribution. This means that for shallow water



$D_{n50}A$ = nominal diameter armor
$D_{n50}F$ = nominal diameter filter
$D_{n50}C$ = nominal diameter core
Structures on Fig. 5a, 5c and 5d have been tested.
The value of P for Fig. 5b has been assumed.

FIG. 5. Permeability Coefficient P

locations the parameter ($H_{2\%}/1.40$) must be used in formulas 1 and 2 (Eqs. 1 and 2) instead of H_s. The factor 1.40 is the ratio $H_{2\%}/H_s$ for a Rayleigh distribution.

3. The wave steepness—The wave steepness, $2\pi H_s/gT_z^2$, should be between 0.005 and 0.06. For a wave steepness greater than 0.06 waves become unstable, and break because of their steepness. This value can therefore be regarded as an upper boundary. The average wave period is used in the formulas defined by the zero up-crossings in the wave signal or by the formula: $T_z = \sqrt{m_0/m_2}$, where m_2 = the second moment of the energy density spectrum. This average period is preferred as it gave the same stability for different spectral shapes, where the peak period, for instance, gave different stability curves.

4. The permeability—The permeability coefficient P was introduced to describe the influence of the permeability of the structure on its stability. Three structures have been investigated. The lower boundary value of P is that given by an impermeable core (clay or sand), see Fig. 5(a). With this impermeable core a value of P = 0.1 was assumed. The upper boundary value of P is that given by a homogeneous structure, consisting only of armor stones, see Fig. 5(d). For this structure a value of P = 0.6 was assumed. The third structure consisted of a two-diameter thick armor layer on a permeable core. The ratio of armor/core stone diameter was 3.2, see Fig. 5(c). For this structure a value of P = 0.5 was assumed. The first and third structure are close to the structures investigated by Hedar (1960, 1986), who also found that the permeability had a large influence on stability.

The value of P for other structures with, for example, more than one layer of stones [Fig. 5(b)] or a thicker armor layer must be estimated from the values established for the three specific structures. The design engineer's experience is obviously important when selecting the value of P.

5. The damage level S—The damage level S is the number of cubic stones with a side of D_{n50}, eroded around the water level within a width of one D_{n50}, see Fig. 4. For a two-diameter thick armor layer the lower and upper damage levels have been assumed to be the values shown in Table

TABLE 1. Lower and Upper Damage Levels for Two Diameter Thick Rock Slopes

cotα (1)	Damage Level $S = A/D_{n50}^2$ Start of damage (2)	Filter layer visible (3)
1.5	2	8
2.0	2	8
3.0	2	12
4.0	3	17
6.0	3	17

1. The definition of start of damage (S = 2 for steep slopes and S = 3 for milder slopes) is the same as that used by Hudson (1958) and Ahrens (1975). The definition of "filter layer visible" may be assumed as failure of the armor layer (although not the immediate failure of the structure).

6. The storm duration—The formulas can be used when the number of waves, or storm duration, is in the range N = 1,000–7,000. For $N > 7000$ the damage tends to be overestimated. The maximum damage is, in fact, the damage found for N = 8,000–9,000.

7. The mass density—The mass density of the stones used in the tests was between 2,000–3,000 kg/m^3 giving relative mass densities Δ in the range 1.0–3.0.

8. Other parameters—Formulas 1 and 2 (Eqs. 1 and 2) do not include parameters for the grading of the stones (riprap or uniform stones) or for the spectrum width or the grouping of the wave signal. The investigation showed no difference in stability between riprap and uniform stones, and also the spectrum shape and grouping of waves did not appear to influence the stability, provided that the average wave period was used and not the peak period.

Deterministic Design Graphs

Design graphs can be drawn using Formulas 1 and 2 (Eqs. 1 and 2). In order to demonstrate the influence of the different parameters the graphs are given for an assumed structure. The properties of this structure are: nominal diameter D_{n50} = 1.0 m; mass density stone ρ_a = 2,600 kg/m^3, i.e., W_{50} = 2,600 kg; mass density water ρ = 1,000 kg/m^3, equivalent to a relative mass density Δ of 1.6; slope angle cotα = 3.0; damage level S = 5 (tolerable damage in 50 yr); Permeability P = 0.5 (permeable core) see Fig. 5(c); and storm duration N = 3,000 waves.

Influence of Wave Height, Period, and Damage Level

Most of the design graphs are shown on $H_s - \xi_z$ plots. The wave height is plotted on the vertical axis and the surf similarity or breaker parameter on the horizontal. The breaker parameter takes into account the influence of the wave period and slope angle. The damage levels S = 2 (start of damage), S = 5 and 8 (tolerable damage), and S = 12 (filter layer visible, failure) have been plotted in Fig. 6. Formula 1 is plotted on the left side of Fig. 6 (plunging waves) and formula 2 on the right side (surging waves).

By using the assumed parameters for the structure given above, the plunging wave curve for S = 5 can be found from formula 1:

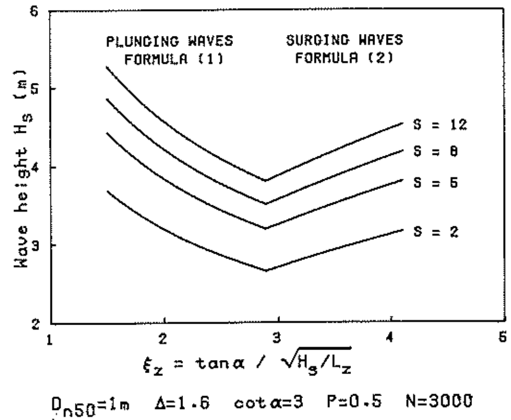


FIG. 6. Influence of Damage Level

$$H_s = 5.43\xi_z^{-0.5} \quad (3)$$

and the surging wave curve from formula 2:

$$H_s = 1.88\xi_z^{0.5} \quad (4)$$

The transition from plunging to surging waves occurs at

$$\xi_z = (6.2P^{0.31}\sqrt{\tan\alpha})^{1/(P+0.5)} \quad (5)$$

This transition (collapsing waves) gives the minimum stability. In the plunging region, wave runup is decisive for stability and in the surging region wave rundown. In the collapsing region both runup and rundown forces are high which causes the minimum of stability.

Influence of Slope Angle

Fig. 7 shows the stability formulas for slope angles with cotα = 1.5, 2.0, 3.0, 4.0, and 6.0. The left side (plunging waves) is given by one curve which means that the breaker parameter is an excellent parameter in the breaking wave region. For slopes gentler than 1:4, surging waves do not occur. For steeper slopes, the minimum decreases, i.e., a lower wave height causes instability, and the transition from plunging to surging waves shifts to the right.

Influence of Permeability

Fig. 8 shows the curves for four values of the permeability coefficient. The value of P = 0.1 (impermeable structure) gives the lower boundary, and the value of P = 0.6 (homogeneous structure) gives the upper boundary. The influence of the wave period for plunging waves (left side of Fig. 8) shows the same trend for all four structures, although a more permeable structure is more stable. A more permeable structure is also more stable for surging waves ($\xi_z > 3.5$), but the stability increases with larger wave periods. The curves are steeper for larger permeability.

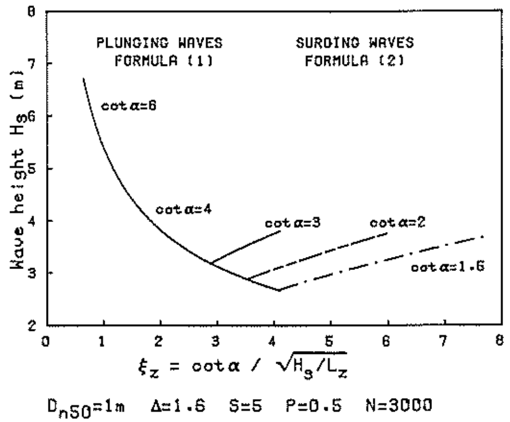


FIG. 7. Influence of Slope Angle

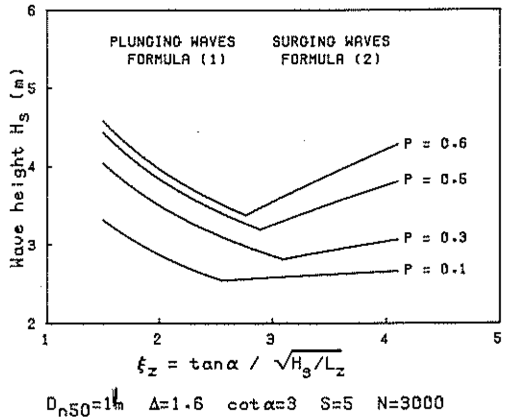


FIG. 8. Influence of Permeability

This phenomenon can be explained in physical terms by the difference in water motion on the slope. For a slope with an impermeable core, the flow is concentrated in the armor layer causing large forces on the stones during rundown. For a slope with permeable core, the water dissipates into the core, and the flow becomes less violent. With lover wave periods (larger ξ_z), more water can percolate and flow down through the core. This reduces the forces and stabilizes the slope.

The stability increases by more than 35% as P shifts from 0.1–0.6 in relation to the wave height. This means a difference of a factor of 2.5 in mass of stone for the same design wave height. This is only caused by a difference in permeability. This aspect is taken into account in the berm breakwater concept (Baird and Hall 1984) where a permeable berm is applied.

Influence of Storm Duration

Fig. 9 shows the damage level of $S = 5$ for different storm durations, i.e., different numbers of waves. For $\xi_z = 2$ and $N = 1{,}000$, this damage level is reached with a wave height of $H_s = 4.3$ m. For a very long storm ($N > 7{,}000$), it is reached with $H_s = 3.5$ m. The storm duration is a parameter that only becomes obvious when testing with random waves. For monochromatic waves, equilibrium is found within 1,000 waves. This means that it is not so easy to use stability formulas developed with monochromatic waves for prototype conditions where the waves are random. It is not simply a matter of replacing H by the significant wave height H_s or even a higher wave height.

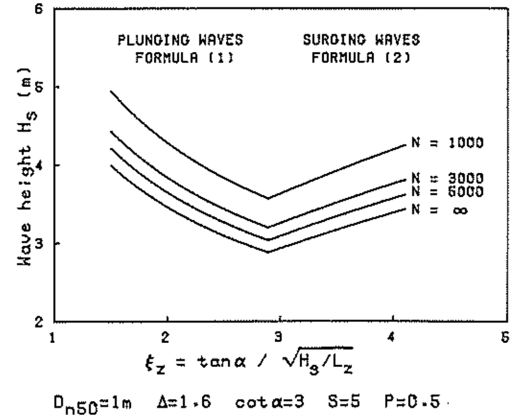


FIG. 9. Influence of Storm Duration

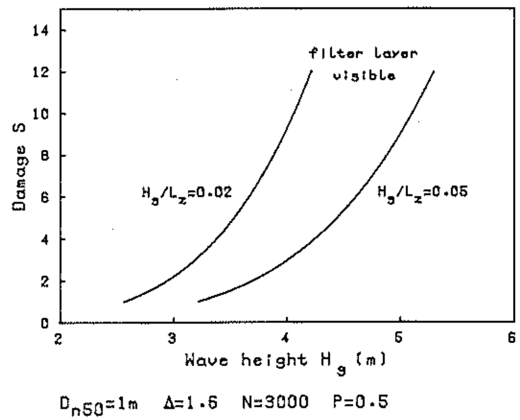


FIG. 10. Damage Curves

Damage Curves

Another graph that can be calculated from the formulas is the damage curve in which the damage is plotted as a function of the wave height. Fig. 10 gives these damage curves for two different values of wave the fictatious steepness, $H_s/L_z = 0.02$ ($S = 0.00907\ H_s^5$, from Eq. 1) and 0.05 ($S = 0.00289\ H_s^5$, also from Eq. 1), where $L_z = gT_z^2/2\pi$.

Conclusions

The graphs (Figs. 6–10) give a good impression of the influence of the governing parameters on stability. Formulas and graph plotting have been programmed on a personal computer. This makes it very easy for the designer to design the armor layer of a rubble mound and to look into the effects of various changes on stability and possibilities for improving design.

Probabilistic Design

The purpose of the design of a revetment or breakwater is to obtain a structure which, during its construction and throughout its intended service life, has a sufficiently low probability of failure and of collapse. In order to achieve the best possible assessment of this, a risk analysis can be performed. The three main elements of the risk analysis are hazard, mechanisms, and consequence.

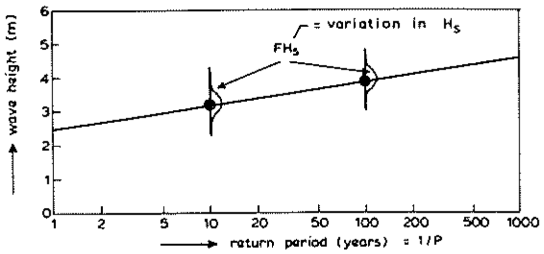
A risk analysis begins with the preparation of an inventory of the hazards and mechanisms. A mechanism is defined as the manner in which the structure responds to hazards. A combination of hazards and mechanisms leads, with a particular probability, to failure or collapse of the structure as a whole or of its components. Here only the mechanism of instability of the armor layer due to wave action will be treated. A probabilistic calculation of the failure probability can be performed using the so-called reliability function Z with regard to the limit state considered. In its simplest form Z = strength−load. Negative values of Z correspond to failure. The probability of failure can thus be represented symbolically as $P[Z < 0]$. The various parameters in an armor layer design (Eqs. 1 and 2) can be allocated to these components as follows: strength = D_{n50}, Δ, $\cot\alpha$, and P; load = H_s, T_z or H_s/L_z, and N.

For armor-layer design, the damage level S is, in fact, equivalent to the failure criterion for the reliability function. This means that probabilities can be calculated for different damage levels. By rearranging formulas 1 and 2, the reliability function can be established. In this case the influence of the wave period is described by the wave steepness H_s/L_z. The reliability function Z for plunging waves is described by

$$Z = S^{0.2} * 6.2P^{0.18} \cot\alpha^{0.5}\, \Delta D_{n50} - (H_s + \mathrm{FH}_s) \cdot \left(\frac{H_s}{L_z}\right)^{-0.25} N^{0.1} \quad \text{.......... (6)}$$

For surging waves:

$$Z = S^{0.2} * P^{-0.13} \cot\alpha^{0.5-P} \Delta D_{n50} - (H_s + \mathrm{FH}_s) \cdot \left(\frac{H_s}{L_z}\right)^{0.5P} N^{0.1} \quad \text{.......... (7)}$$



$P(H_s) = P[H > H_s] = \exp[-(H_s - 2.5)/0.3]$

FIG. 11. Long-Term Distribution of Wave Height H_s with Its Variation FH_s

where FH_s = a parameter to account for the uncertainty of the wave height (see Fig. 11).

The long-term distribution of the wave height can be described by an exponential (Weibull) function:

$$P(H_s) \equiv P[H \geq H_s] = \exp\left\{-\left[\frac{(H_s - C)}{B}\right]^{\gamma}\right\} \quad \text{.......... (8)}$$

in which C = the background noise level or lower-boundary; B = the scale parameter; and γ = the shape parameter. The uncertainty of this long term distribution is given by the FH_s parameter in Eqs. 6 and 7 which has an average value of zero and has a normal distribution. For the example given here, $C = 2.5$, $B = 0.3$, and $\gamma = 1.0$. This long-term distribution of the wave height is shown in Fig. 11, together with the parameter FH_s. The 1/50 yr wave height in this case, calculated using Eq. 8 with $P = 1/50$, is 3.67 m. The values for the average and the standard deviation used for the computations are shown in Table 2. The average values are the same as in deriving deterministic design curves.

The parameters a and b in this table give the uncertainty of the stability formulas; a = the coefficient 6.2 in Eq. 1; and b = the coefficient 1.0 in Eq. 2. The standard deviations of the stability formulas were derived from the

TABLE 2. Parameters Used for Level II Probabilistic Computations

Parameter (1)	Distribution (2)	Average (3)	Standard deviation (4)
D_{n50} (m)	Normal	1.0	0.03
Δ	Normal	1.6	0.05
$\cot\alpha$	Normal	3.0	0.15
P	Normal	0.5	0.05
N	Normal	3,000	1,500
H_s	Weibull	$B = 0.3$	$C = 2.5$
FH_s	Normal	0	0.25
H_s/L_z	Normal	0.04	0.01
a (Eq. 1)	Normal	6.2	0.4
b (Eq. 2)	Normal	1.0	0.08

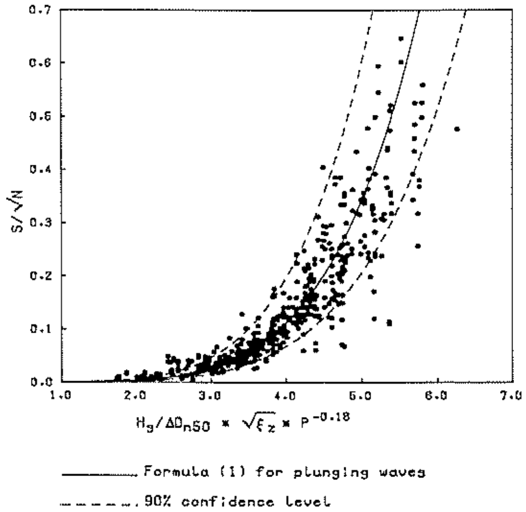


FIG. 12. Formula 1 for Plunging Waves with Test Results

variation of all the model test results around the curves of the formulas. Fig. 12 gives formula 1 with the model test results and the 90% confidence level. From this figure a standard deviation of $\sigma = 0.4$ was established. The standard deviation for formula 2 was found to be 0.08. The uncertainty of formulas 1 and 2 is due to curve-fitting and to the random behavior of armor stones that will also occur in nature.

The level II first-order second-moment (FOSM) with approximate full distribution approach (AFDA) method was used for the computations. General references on this aspect are Thoft-Christensen and Baker (1982) and Hallam et al. (1977). These computations were performed for several damage levels S. The results give the probability of occurrence of that damage level in one year. These probabilities per year are small and are shown in Fig. 13. The horizontal axis shows the damage level S, and the vertical axis shows the probability of exceedance of these damage levels in one year. The probability of exceedance in one year of the damage level S = 4, for instance, is equal to about 2%.

The probability of exceedance for an X-year period can be obtained using:

$$P[Z < 0; X \text{ yr}] = 1 - (1 - P[Z < 0, 1 \text{ yr}])^X \quad (9)$$

Results, derived from Fig. 13 using Eq. 9, are shown in Fig. 14. Curves are drawn for three lifetimes (X = 20, 50, and 100 years). From this figure it follows that the damage level S = 2, which means start of damage, will certainly occur in a lifetime of 50 years. Tolerable damage (S = 5–8) in the same lifetime will occur with a probability of 0.2–0.5. The probability that failure (filter layer visible) will occur in a lifetime of 50 years is less than 0.1.

Such probability curves can be used to make a cost optimization for the breakwater during its lifetime. This procedure is discussed in Nielsen and

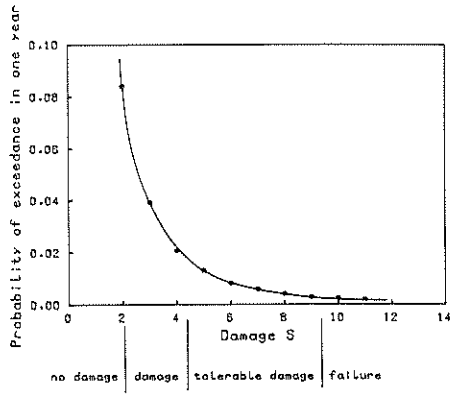


FIG. 13. Probability of Exceedance of Damage Level S in One Year

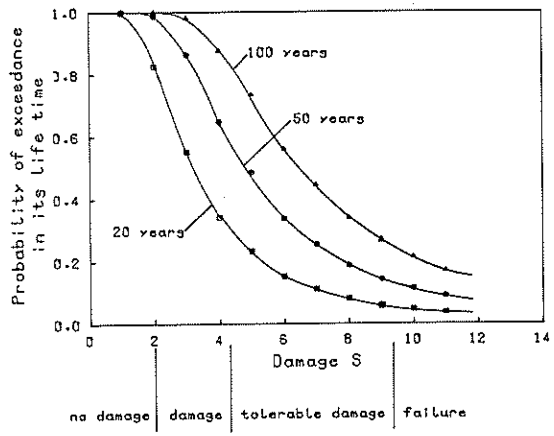


FIG. 14. Probability of Exceedance of Damage Level S in Lifetime of Structure

Burcharth (1983) and Le Mehaute (1985). Probabilistic design taking into account other failure mechanisms for a breakwater are described in the CIAD report (1985).

Conclusions

New stability formulas have been derived from the results of extensive model investigations. These formulas have been used to develop design graphs that show the influence of various parameters. The formulas have been rewritten in the form of reliability functions. Level II computations were performed, which resulted in graphs showing the probability of exceedance of different damage levels during the lifetime of the structure.

Both deterministic and probabilistic design procedures have been used in an example. The same kind of computations can be performed for other structures, using this example as a reference.

Appendix I. References

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Appendix II. Notation

The following symbols are used in this paper:

A = erosion area in a cross section;
a = coefficient 6.2 in Eq. 1, used as a stochastic variable;
AFDA = approximate full distribution approach;
B = scale parameter in exponential function;
b = coefficient 1.0 in Eq. 2, used as a stochastic variable;
C = lower boundary in exponential function;
D_{n50} = nominal diameter $(W_{50}/\rho_a)^{1/3}$;
FH$_s$ = uncertainty parameter of wave height H_s;
FOSM = first-order second-moment approach;

g = acceleration due to gravity;
H_s = significant wave height;
H_{10} = average of highest 10% of waves;
L_z = deep water wave length $(gT_z^2/2\pi)$;
m_0 = zeroth moment of energy density spectrum;
m_2 = second moment of energy density spectrum;
N = number of waves (storm duration);
N_s = stability number $(H_s/\Delta D_{n50})$;
P = permeability coefficient;
$P[Z < 0]$ = probability of failure;
S = damage level (A/D_{n50}^2);
T_z = average wave period;
W_{50} = 50% value of mass distribution curve;
X = life time of the structure (years);
Z = reliability function;
α = slope angle;
γ = shape parameter in exponential function;
Δ = relative mass density $(\rho_a/\rho - 1)$;
ξ_z = surf similarity parameter for random waves $(\tan\alpha/\sqrt{H_s/L_z})$;
ρ_a = mass density of water;
ρ = mass density of stone; and
σ = standard deviation.