

CHAPTER 126

PROBABILISTIC CALCULATIONS OF WAVE FORCES ON VERTICAL STRUCTURES

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ABSTRACT

In the past wave forces on vertical structures have been measured in a number of site specific projects at the Danish Hydraulic Institute and Delft Hydraulics. For nine selected cases, the data on forces and moments were re-analyzed, leading to an expression for reliability of Goda's formula for calculation of forces and moments on vertical breakwaters. Secondly, probabilistic level II design calculations were made using Goda's formula with the found reliability. It appears that the reliability of this formula has by far the largest influence on the probability of failure. Finally the influence of model tests on the probability of failure was studied. In that case the wave height has the largest influence on stability, which is usual for most coastal structures designs.

INTRODUCTION

Design of vertical breakwaters has for instance to take into account hydraulic, geotechnical and structural aspects. The wave forces exerted on a vertical structure depend on characteristics of the incident waves, type of structure, elasticity of the structure, air enclosure and the entrainment of dissolved air and foreshore characteristics

In a deterministic design approach, vertical breakwaters are designed based upon characteristic values of the load determining parameters. A safety factor is then introduced to allow for uncertainties. However, all parameters which are important

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for the wave loads on and the strength of a hydraulic structure are of a stochastic nature, ie a probability distribution can be assigned to each value of these parameters. Since the last decades methods for probabilistic design of hydraulic structures have been developed, taking into account the stochastic nature of the load determining parameters.

The formulae of Goda (1985) for wave forces on vertical structures is used worldwide. The distribution of wave pressures on a vertical structure can be calculated based on knowledge of structure geometry, seabed characteristics and wave parameters in front of the structure, see Figure 1, taken from Goda (1985).

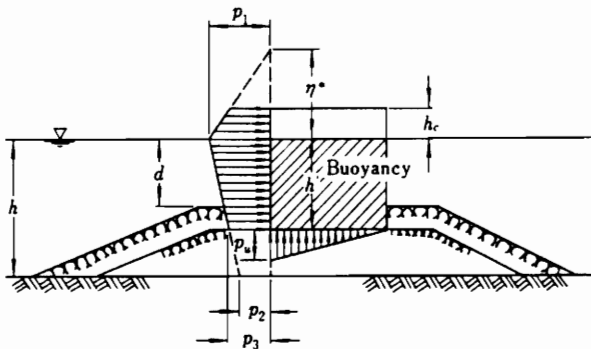


Figure 1 Distribution of wave pressures on a vertical structure, Goda (1985)

The design wave parameters are the maximum wave height in front of the structure, H_{\max} , and the corresponding wave period taken as the significant wave period, T_s (which is close to the peak wave period, T_p). Goda (1985) states that: " H_{\max} is the mean of the heights of the waves included in 1/250 of the total number of waves, counted in descending order of height from the highest wave. This definition yields the approximate relation $H_{\max} = 1.8 H_{1/3}$ outside the surfzone". The formulae for pressures, total forces and moments, and definitions of symbols given in Fig. 1 can be found in Goda (1985) pp 115-119.

The present paper describes a re-analysis of model tests on various vertical structures carried out at the Danish Hydraulic Institute and Delft Hydraulics. Later two cases were added from CEDEX-CEPYC, Spain, and ENEL-CRIS, Italy. The reliability of Goda's formulae has been established by use of these practical cases by making comparisons of calculations and measurements. In a second stage, these data have been used for probabilistic level II calculations.

RE-ANALYSIS

A total of eleven cases has been re-analyzed with respect to wave forces and moments on vertical structures. Details on caisson geometry, foreshore slopes, wave conditions and horizontal forces were stored in a database as presented in Juhl and Van der Meer (1992). The analysis of horizontal forces is also described in Juhl and Van der Meer (1992) with a summary in Van der Meer et al. (1992). Vertical forces and overturning moments were treated later by Bruining (1994).

The analyzed cases have been divided in three categories: vertical superstructure, inclined superstructure, and curved superstructure. In Figure 2, an example of each of these three categories is shown.

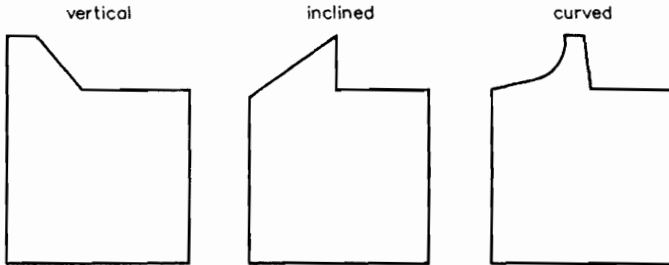


Figure 2 Examples of different superstructures

The significant and maximum wave heights in front of the vertical structure were calculated by Goda's formulae for transformation and deformation of random sea waves, Goda (1985) pp 71-87. Only total wave forces, measured by means of strain gauge based measuring equipment (eg a measuring frame or a dynamometer), are considered. This means that local wave impacts are not treated, as they cannot be measured by this measuring technique. It was assumed, however, that very fast local impacts would not influence the stability of a caisson and were therefore not interesting.

In all cases, wave trains with lengths of 1000-3000 waves were considered. Based on the recordings the horizontal and vertical forces and horizontal and vertical overturning moments at the heel of the caisson were tabulated, with exceedance frequencies of 0.1%, 0.4%, 1%, 2% and 5% respectively. In Figures 3-5 the measured horizontal forces, $F_{0.4\%}$, are compared with the horizontal forces calculated by Goda's formula, F_{Goda} . The cases with a vertical superstructure have been plotted in Figure 3, and the cases with an inclined or curved superstructure have been plotted in Figure 4. Due to the various structure geometries, foreshore slopes, wave characteristics, etc., a significant scatter in the measured wave forces was found.

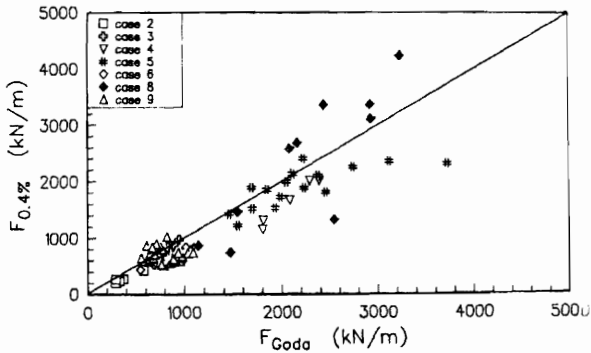


Figure 3 Measured and calculated horizontal wave forces; vertical superstructures

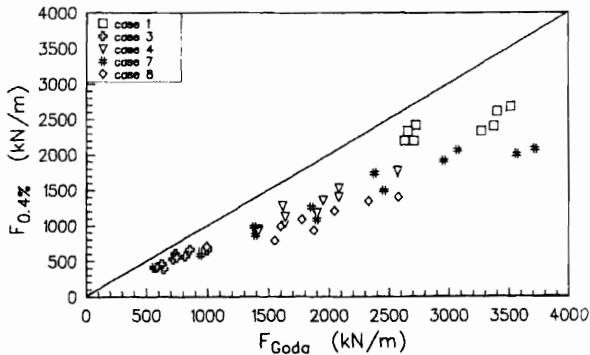


Figure 4 Measured and calculated horizontal wave forces; inclined and curved superstructures

From Figure 4, it is clear that for inclined and curved superstructures, the wave forces calculated by Goda's formula are much higher than the measured forces. In the major part of those cases, the ratio between the calculated and measured force is in the order of 1.4-1.6. For inclined and curved superstructures, the maximum force on the superstructure occurs later than the maximum horizontal force on the vertical front. This phase difference in the forces led to the following modification of Goda's formula for inclined and curved superstructures:

The crest height should be determined at the transition from the vertical front to the inclined or curved superstructure.

The horizontal forces, F_{Goda} , were re-calculated with a reduced crest height in accordance with the above-mentioned modification to Goda's formula. The results are presented in Figure 5 and it is found that the ratio's between calculated and measured forces are in the same order as found for the cases with a vertical superstructure.

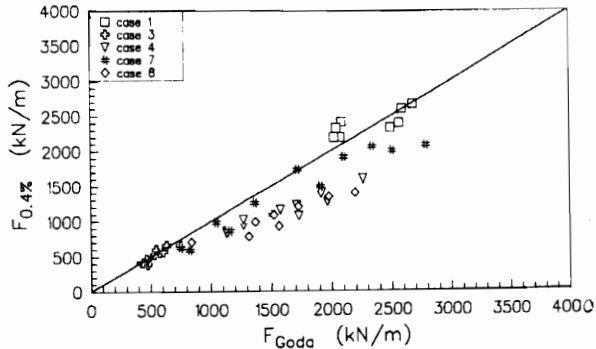


Figure 5 Measured and calculated horizontal wave forces; inclined and curved superstructures, ignoring the inclined or curved superstructure in the Goda force calculations

The average ratio between measured ($F_{0.4\%}$) and calculated (F_{Goda}) forces and moments was based on 134 data sets for horizontal forces and moments; 31 data sets were used for the uplift forces and moments and were treated in the same way as the horizontal forces. The average values of these ratio's and the standard deviations, assuming a normal distribution, are given in Table 1.

		Ratio measured/calculated	Average	Standard deviation
Horizontal force	F_h	r_{Fh}	$\mu = 0.83$	$\sigma = 0.25$
Horizontal moment	M_h	r_{Mh}	$\mu = 0.75$	$\sigma = 0.40$
Vertical force	F_b	r_{Fb}	$\mu = 0.71$	$\sigma = 0.25$
Vertical moment	M_b	r_{Mb}	$\mu = 0.67$	$\sigma = 0.37$

Table 1 Comparison between measured and calculated forces and moments
Measured = 0.4% exceedance; calculated = Goda

In all cases the ratio is smaller than one, which means an overprediction by the Goda formulae. The most important conclusion is that the standard deviations are large. The variation coefficients, σ/μ , amount to 30-50%! It can be concluded that the Goda method gives only a rough estimation of the forces and moments.

Through a more in depth description of each of the cases, the following observations were made:

- Goda's formula is valid for caissons founded on a rubble mound berm well above the seabed. In a number of the tested cases, the caisson was founded at the seabed level. The results of some of these cases give the impression

that Goda's formula over-predicts the horizontal forces when the caisson is founded at the same level as the seabed;

- no general conclusions could be made on the influence of wave breaking on the foreshore or on the wave period (or wave steepness).

EXCEEDANCE CURVE FOR THE HIGHEST FORCES

Figure 6 shows an example of measured exceedance curves for the horizontal force. Through the five tabulated measured horizontal, but also vertical forces, with exceedance values of 5%, 2%, 1%, 0.4% and 0.1%, a two-parameter Weibull distribution was fitted for each test run (95 in total). This analysis resulted in an average shape parameter of 2.1 which corresponds closely to a Rayleigh distribution for the higher wave forces. The reliability of this factor 2.1 could be described by a standard deviation of 0.72. The average value for the vertical forces amounted to 2.35 with a standard deviation of 0.77. The conclusion was that, considering the large scatter, the distribution of the highest wave forces can be described by a Rayleigh distribution with a shape parameter of 2:

$$R(F) = e^{-\left(\frac{F}{a}\right)^2} \quad (1)$$

where $R(F)$ is the exceedance probability and a the scale parameter. Results from using the Goda method were compared with measured 0.4% exceedance values. These ratio's have been given in Table 1. The Goda force is not equal to $1/250 = 0.4\%$, but equal to the average of the highest $1/250$ -th part of the forces. Now the shape of the force distributions has been found to be a Rayleigh distribution one can calculate the ratio between the average of the highest $1/250$ -part and the 0.4%. This ratio amounts to 1.084. With this factor the actual comparison between Goda and measured forces can be given as the average ratio's presented in Table 1, multiplied by 1.084. Table 2 gives the result.

		Ratio measured/calculated	Average	Standard deviation
Horizontal force	F_h	r_{Fh}	$\mu = 0.90$	$\sigma = 0.25$
Horizontal moment	M_h	r_{Mh}	$\mu = 0.81$	$\sigma = 0.40$
Vertical force	F_b	r_{Fb}	$\mu = 0.77$	$\sigma = 0.25$
Vertical moment	M_b	r_{Mb}	$\mu = 0.72$	$\sigma = 0.37$

Table 2 Comparison between the Goda method and measured values, both based on the average of the highest $1/250$ -th values

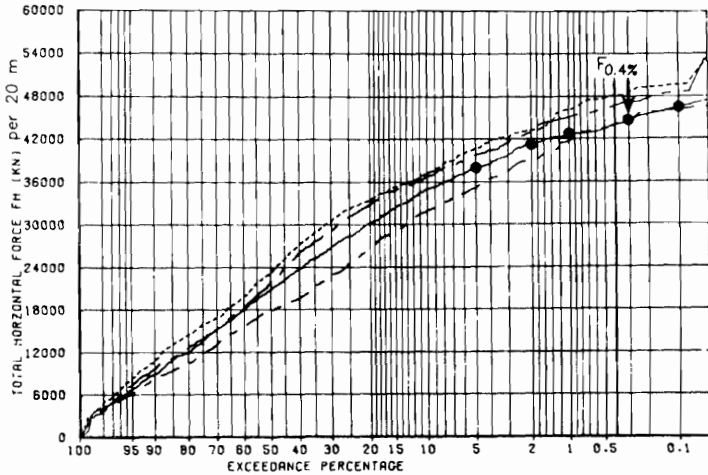


Figure 6 Example of measured distributions of horizontal forces

The most important force for stability is normally the horizontal force F_h . Based on the selected cases Goda overestimates this force by 10%. During a lecture Takayama (1994) gave the Japanese experience: based on 66 cases the average ratio amounted to $\mu = 0.91$ with a standard deviation of 0.19. These values are in good agreement with those found for the horizontal force F_h , see Table 2.

The scale parameter in Equation 1 can be based on the Goda formula and on the found bias and reliability, given in Tables 1 or 2. With $R(F_{0.4\%}) = 0.004$ substituted in Equation 1 the scale parameter a can be replaced using $F_{0.4\%}$:

$$R(F) = e^{-\left(\frac{2.35 F}{F_{0.4\%}}\right)^2} \tag{2}$$

With the factors r (r_{Fh} ; r_{Fb} ; r_{Mh} ; r_{Mb} , given in Table 1), which includes $F_{0.4\%}$, the formula becomes:

$$R(F) = e^{-\left(\frac{2.35 F}{r F_{0.4\%}}\right)^2} \tag{3}$$

Equation 3 can be seen as a design formula for the exceedance curve of the highest forces, based on Goda's method.

In reality the maximum wave force is related to the maximum number of waves during the sea state considered and not to the 0.4% or 1/250 wave only. Taking into account the actual maximum wave force based on the actual storm duration, a second factor, r_N , can be introduced:

REFERENCE CASE FOR PROBABILISTIC CALCULATIONS

One test in one of the nine cases described in Juhl and Van der Meer (1992) will be taken as a reference (case 1, test 4F). Figure 8 gives a cross-section of the caisson. The parameters that are required for a calculation with the Goda formula are:

$$\begin{aligned} H_0' &= 8.0 \text{ m (once per 50 years storm)} \\ T_p &= 15.4 \text{ s} \\ \text{Storm duration: } &8 \text{ hours (} N = 2550 \text{)} \\ h_{\text{sea}} &= 30.5 \text{ m} \\ h &= 19 \text{ m} \\ h_c &= 8 \text{ m, but inclined superstructure: } h_c = 1 \text{ m} \\ d &= 19 \text{ m} \\ \tan m &= 0.002 \end{aligned}$$

Furthermore a weight, W , of 5500 kN/m length and a friction coefficient, f , of 0.7 are assumed. Based on later communications with Takayama (1994) the Japanese experience on the friction coefficient can be summarized as follows. The design value is $f = 0.6$. Based on 42 cases on nearly prototype scale the average friction factor amounted to $\mu = 0.64$ with a standard deviation of $\sigma = 0.16$.

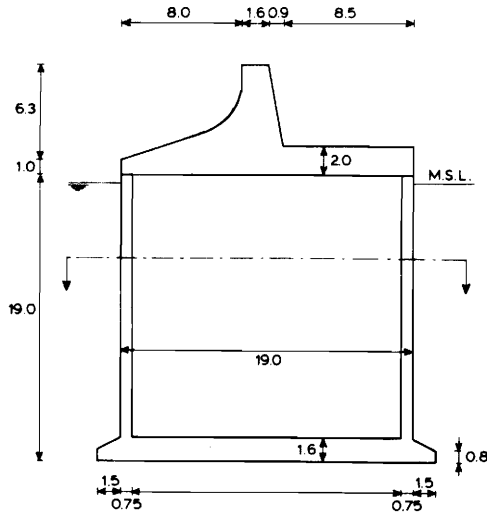


Figure 8 Cross-section of the caisson used for calculations

PROBABILISTIC CALCULATIONS

In a probabilistic approach a reliability function, Z , should be given, which is in fact a design formula or design process. The two main failure mechanisms for a caisson are sliding and overturning. The reliability function for sliding becomes:

$$Z = (W - r_{Fb} r_N F_{b(Goda)}) f - r_{Fh} r_N F_{h(Goda)} \quad (6)$$

The first term describes the weight minus uplift force, multiplied by the friction coefficient. This gives the total friction resistance. The second term gives the maximum horizontal force. $F_{b(Goda)}$ is the vertical force on the base of the caisson and $F_{h(Goda)}$ is the horizontal force on the front side, both calculated by the Goda method described in Goda (1985) and depending on a large number of parameters.

A probabilistic approach with Equation 6 gives the probability that the caisson will slide during a (design) sea state. All calculations were made with a level II first-order second-moment (FOSM) with approximate full distribution approach (AFDA) method. General references on this aspect are Thoft-Christensen and Baker (1982), Hallam et al. (1977) and PIANC (1993). Calculations have been made for three cases: one for the design event, one for the life time of the structure and one including the results of physical model tests.

Case 1: Calculations for the Design Event

In order to show the influence of the uncertainty of the Goda formula on the probability of failure a few different calculations have been made. One calculation has been made with only r and r_N as stochastic variables, another with f and W also as stochastic variables and finally one also including all the parameters in F_{Goda} as stochastic variables. The mean values and standard deviations of the normal distributions used in the calculations are shown in Table 3. The factor r_N is described by the number of waves, N , see Equation 4.

The probabilities of failure, $P(f)$, for the three calculations were 0.062, 0.086 and 0.097, respectively. These are the probabilities of failure during the (design) sea state of 1/50 years. All three probabilities are close, which means that the reliability of the Goda formula, by means of r_{Fb} and r_{Fh} , have by far the largest influence. In fact the influence of r_{Fh} , the reliability of the horizontal force, on the probability of failure amounted to 99%, 73% and 65% in the three calculations, respectively.

Parameter	Mean	Standard deviation	
r_{Fh}	0.83	0.25	calculation 1
r_{Fb}	0.71	0.25	
N	2550	127 (5%)	
f	0.7	0.1	calculation 2
W (kN/m)	5500	165 (3%)	
H_0' (m)	8.0	0.4 (5%)	calculation 3
T_p (s)	15.4	1.54 (10%)	
h' (m)	19	0.57 (3%)	
h_c (m)	1	0.03 (3%)	
h_{sea} (m)	30.5	0.915 (3%)	
d (m)	19	0.57 (3%)	

Table 3 Parameters and values used for calculations.

Case 2: Calculations for the Life Time of the Structure

More design information is obtained when not one (design) sea state is considered, but the whole wave climate by means of an extreme distribution for the wave heights. Taking the 1/50 years wave height as a reference (which was used for testing), such an extreme distribution can be established by means of an exponential distribution:

$$R(H_s) = e^{-\frac{(H_s - 5.1)}{0.51}} \quad (7)$$

In this case the once per year wave height is 5.1 m ($R(H_s) = 1$) and the 1/50 years wave height becomes 8.0 m ($R(H_s) = 0.02$).

With Equation 7 a probabilistic calculation gives the probability of failure per year instead of during the (design) sea state. Calculation 3 of case 1 above, where all parameters were treated stochastically, has been performed again, but now with the exponential distribution of the wave height (Equation 7) instead of the normal distribution for the 1/50 years wave height. This results in a probability of failure of 0.018 per year. The influence of r_{Fh} on the probability of failure amounted to 55% and of the wave height H_s to 17%.

The probability of exceedance for an X-year period can be obtained using:

$$P[Z < 0; X \text{ yr}] = 1 - (1 - P[Z < 0; 1 \text{ yr}])^X \quad (8)$$

With the above result of a probability of failure per year of 0.018 a graph can be drawn with the probability of failure as a function of the life time of the structure. The upper solid line in Figure 9 gives the result of above calculations. The proba-

bility of failure for a life time of 50 years is 0.60, considerably higher than for the 1/50 years sea state only (0.097). This is probably due to the fact that wave heights lower than the 1/50 years wave height increase the probability, but that also higher wave heights (with a lower probability of occurrence) increase the total probability.

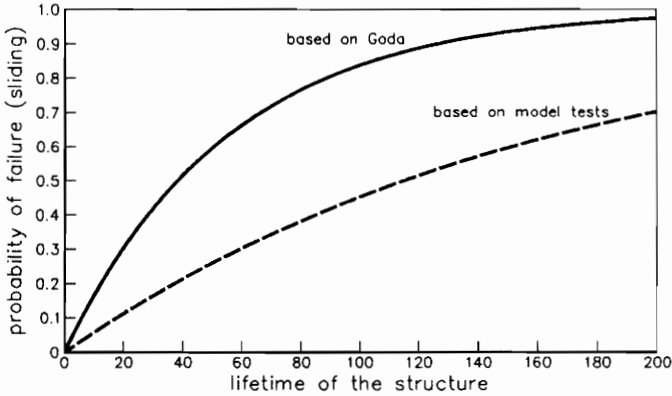


Figure 9 Probability of failure as a function of the life time of the structure

Case 3: Use of Results of Physical Model Tests

Until now the calculations were done for the Goda formula only, including the uncertainties of all the parameters. The large variation of the Goda formula by means of $\sigma(r)$ has the largest influence on the failure probability. This variation is due to the large variety of structure geometries and foreshores that were present in the nine selected cases (see Juhl and Van der Meer (1992) and Bruining (1994)). This large variation can be eliminated by performance of physical model tests. In that case forces are measured for the specific structure geometry including all effects of the foreshore.

With respect to Equation 6 it means that the exact values of r_{Fh} and r_{Fb} are known (the bias) and the scatter of r_{Fh} and r_{Fb} is much smaller than $\sigma(r) = 0.25$. Further the ratio r_N (Rayleigh distribution or not) is also known. The same calculations can be done as in case 2, but now with r_{Fh} and $r_{Fb} =$ measured value, $\sigma(r) = 0.05$ (assumed) and $r_N =$ measured value.

The following factors were determined by model tests. Between brackets the previous values, based on the Goda method only, are also given.

r_{Fh}	= 0.88	$\sigma = 0.05$	(0.83; $\sigma = 0.25$)
r_{Fb}	= 0.67	$\sigma = 0.05$	(0.71; $\sigma = 0.25$)
$r_N (F_h)$	= 1.21	$\sigma = 0.05$	(1.19; $\sigma = 0.05$)
$r_N (F_b)$	= 1.11	$\sigma = 0.05$	(1.19; $\sigma = 0.05$)

The calculations of case 2 were repeated with the new factors given above. The probability of failure per year amounted now to 0.006, a factor three lower than for case 2. The influence of the most important parameters for both cases (with and without model tests) on the probability of failure is given in Table 4.

With model tests	Without model tests
Probability of failure $p = 0.006$	Probability of failure $p = 0.018$
Influence of	
r_{Fh} : 2%	r_{Fh} : 55%
f : 33%	f : 15%
H_s : 45%	H_s : 17%
T_p : 16%	T_p : 10%

Table 4 Influence of main parameters on the probability of failure, with and without model tests

The influence of the wave height amounted now to 45% (this was 17% in case 2) and becomes the most important parameter, which is usual, see PIANC (1993).

With Equation 8 and the probability of failure per year of 0.006 the lower dashed line in Figure 9 was calculated. The probability of failure for a life time of 50 years becomes now 0.26 (this was 0.60 in case 2). Figures like Figure 9 can be used by designers. They have to decide which probabilities of failure to accept.

CONCLUSIONS

The performed re-analysis of the data on wave forces on vertical structures for eleven selected cases gave the following results and conclusions:

- An inclined or curved superstructure results in much lower wave forces than a vertical superstructure. It was found that by ignoring the inclined/ curved superstructure in Goda's formula for horizontal forces (i.e. the crest height is determined as the transition from the vertical front to the inclined/curved superstructure) the force ratio's (calculated/measured) were in the same order of magnitude as for completely vertical structures
- In general, the horizontal forces calculated by Goda's formula are about 10% higher than the corresponding measured forces. This is in agreement with Japanese experience. However, a considerable scatter (standard deviation 0.25 on a ratio between measured and calculated force) is present due to the site specific differences, eg caisson geometry and foreshore slopes.
- The results indicate that Goda's formula over-estimates the horizontal wave forces on a caisson founded at the same level as the bottom of the foreshore, i.e. in the absence of a traditional rubble mound foundation.

Probabilistic calculations gave the following main results and conclusions:

- The Goda formula gives in fact a good (average) estimate of the maximum horizontal wave force when the sea state has a duration of some hours, including about 2000 - 3000 waves.
- Probabilistic calculations show that the reliability of (scatter around) the Goda formula by means of the factor r_{Fh} has by far the largest influence of all parameters on the probability of failure. Model tests are therefore advised in all cases and these will decrease this scatter and the influence on the failure probability.
- Design graphs of failure probability (of sliding or overturning) versus desired life time of the structure can be given as a result of probabilistic calculations.

ACKNOWLEDGEMENTS

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