

# Statistical Characterisation of Extreme Overtopping Wave Volumes

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## Summary

The starting point for this paper is the recently developed formula for the shape parameter of the Weibull distribution of overtopping wave volumes at a coastal defence. This shape factor  $b$  increases with decreasing relative freeboard and was developed strictly for smooth and impermeable structures like a dike or levee. Data on overtopping wave volumes are available for rubble mound structures, but the distributions of the volumes have never been analysed jointly with smooth-slope distributions and have never been compared with the new formula for smooth and impermeable dikes. This paper describes the analysis of the Weibull  $b$ -value for conventional rubble mound breakwaters as well as for low crested structures with the crest at or just above the water level. These data are then compared with the trend for smooth structures. It is concluded that rubble mound structures show more scatter in the  $b$ -value than smooth impermeable structures, but the same trend is found. The combined data make even more sense if the  $b$ -value is related to relative discharge instead of relative freeboard because the effects of slope angle and wave steepness are implicitly included. An improved prediction formula is proposed representing permeable and impermeable structures.

## Introduction

In the design of coastal defences and in the estimate of their vulnerability a key aspect is the realistic prediction of the characteristics of the overtopping waves. In fact hydrodynamic forces on landward-side slopes largely depend on the distribution of instantaneous overtopping wave volumes, flow thicknesses and flow velocities (Van der Meer et al., 2010; Hughes et al., 2012). Overtopping wave volumes have been successfully approximated by a Weibull distribution, whose shape factor appears to be larger for very large overtopping and certainly for wave overtopping combined with overflow (Hughes and Nadal, 2009; Victor, et al., 2012). A larger shape factor results in lower maximum overtopping wave volumes, keeping the mean overtopping volume the same.

The percent exceedance distribution of overtopping wave volumes is given by (Hughes et al., 2012):

$$P_{V\%}(V_i \geq V) = \exp \left[ - \left( \frac{V}{a} \right)^b \right] \quad (1)$$

where  $P_{V\%}$  is the percentage of wave volumes that will exceed the specified volume ( $V_i$ ). The two parameters of the Weibull distribution are the non-dimensional shape factor,  $b$ , that helps define the extreme tail of the distribution and the dimensional scale factor,  $a$ , that normalizes the distribution. At present the relationship of the Weibull shape factor  $b$  is only valid for smooth and impermeable structures like dikes and levees. The relationship is given as  $b$  versus  $R_c/H_{m0}$  to describe the distribution of overtopping wave volumes (Hughes et al. 2012). Wave-by-wave overtopping data on rubble mound structures, however, are available from various sources; the following will be used in this paper:

- 2D and 3D data for low-crested structures with the crest around still water level (from DELOS project);
- CLASH (2004): various armour type under design conditions.

The main objective of the paper is: how does the Weibull shape factor  $b$  for rough and permeable structures compare with those of smooth and impermeable structures? Predictive formulae may also give a good estimation of maximum overtopping volumes, which often has consequences for design purposes at the area of the crest of rubble mound structures or seawalls. Moreover, for simulation of overtopping with the Wave Overtopping Simulator (Van der Meer et al., 2010) a correct distribution is required.

## Data on Rubble Mound Structures

The data set of Low Crested rubble-mound Breakwaters (LCBs) is composed of 2D tests performed at the University of Firenze, IT (Zanuttigh and Martinelli, 2008) and 3D tests carried out at Aalborg University, DK (Kramer et al., 2005), and at the Polytechnic of Bari, IT (Martinelli et al., 2006). These tests include structures with narrow and wide crests; milder and steeper off-shore slopes; emerged and zero-freeboard conditions.

The structures tested in Firenze were characterised by different crest widths ( $B_c=0.2, 0.4$  and  $0.5$  m) and seaward slopes (1:2 and 1:4). They were made of quarry rock material of fairly homogeneous size, mass density of  $2600 \text{ kg/m}^3$ , nominal diameter  $D_{n50}=0.031$  m and mean porosity  $n=0.42$ . The layout tested in the Aalborg wave basin consisted of two detached breakwaters forming a rip channel in the middle. The structures were composed of an armour layer of quarry rocks with density of  $2650 \text{ kg/m}^3$  on a core with slopes 1:2. The structures tested in Bari consisted of two horizontal layers, the foundation ( $D_{n50}=0.030$  m) and the structure itself ( $D_{n50}=0.045$  m), which was  $0.11$  m high and  $0.30$  m wide at the crest.

In all cases, measurements of the surface elevation were performed by means of resistive wave gauges placed over the structure crest (in the 3D cases, Kramer et al., 2005; Martinelli et al., 2006) or inside the structure (in the 2D case, Zanuttigh and Martinelli, 2008). Furthermore, incident and transmitted waves were obtained from two arrays of wave gauges placed in front and behind the structure.

Specific tests on overtopping were performed in the CLASH (2004) project on conventional rubble mound breakwaters and various armour types were investigated. Wave-by-wave overtopping was measured in an overtopping box on a weighing scale. The overtopping was taken at  $3D_{n50}$  from the edge of the crest. Results are described in Bruce et al., 2009, as well as in the EurOtop, 2007, Chapter 6, with the main focus on average overtopping and providing the correct roughness influence factor for each type of armour unit. Previously, an analysis of  $b$ -values was made, but the conclusion was rather disappointing: "*From the graph and perhaps against expectations, it appears that the single event overtopping volume response of single- and two-layer systems is indistinguishable*" (Bruce et al., 2009). It should be noted that the  $b$ -value was found by fitting on the whole range of measured overtopping wave volumes, which may place undue weight on smaller wave volumes that results in a poor fit to the more important larger volumes. Therefore, reconsideration of the rubble-mound data started with a new fitting, as described farther on in this paper.

## The Procedure for Data Elaboration at Low-Crested Breakwaters

At LCBs with the crest around the water level, it is not possible to measure the overtopping wave volume directly and a method was needed to calculate the volumes, using the records of the wave gauges. A zero down-crossing procedure has been applied to the data acquired at the wave gauges over the crest. The wave front velocity  $c$  was evaluated from the time delay between the waves travelling at the wave gauges placed at known distance over the crest. The overtopping event is schematised as a simple progressive wave that travels with constant front velocity  $c$  and is characterised by a profile  $\zeta(x-ct)$ . Under these hypotheses,

$$\frac{\partial}{\partial t} + c \frac{\partial}{\partial x} = 0 \text{ for any quantity;}$$

from mass balance, the following relation can be thus derived

$$\frac{\partial \zeta}{\partial t} + \frac{\partial q}{\partial x} = \frac{\partial}{\partial x} (q - c \cdot \zeta) = \frac{\partial}{\partial t} (q - c \cdot \zeta) = 0 \quad (2)$$

in which  $q$  is the discharge per unit width and  $\zeta$  is water depth over barrier crest level. Eq. (2) can be rewritten as

$$q - q_{trough} = c(\zeta - \zeta_{trough}) \quad (3)$$

where the subscript 'trough' denotes the discharge per unit width and water depth over barrier crest level at troughs. In case of emerged or zero-freeboard structure, the crest is dry  $\zeta_{trough}=0$  and at the flux at trough is zero  $q_{trough}=0$ . Eq. (3) becomes

$$q = c \cdot \zeta \quad (4)$$

that allows derivation of the overtopping discharge per unit length associated to each wave. The overtopping discharge can then be calculated by integrating the contributions of each wave.

In the case of rubble mound structures, some water is lost by percolation in the rubble mound; and therefore, not all of the overtopping waves identified at the wave gauges placed at the seaward edge result in a corresponding signal at the other wave gauges placed over the crest. When this occurs, the wave front velocity is assumed to be undefined; and therefore, the contribution of the overtopping wave at the off-shore edge is disregarded.

## Wave Overtopping Discharge

For sloping structures like dikes or levees EurOtop (2007) gives the following design formulae:

$$\frac{q}{\sqrt{g \cdot H_{m0}^3}} = \frac{0.067}{\sqrt{\tan \alpha}} \gamma_b \cdot \xi_{m-1,0} \cdot \exp\left(-4.75 \frac{R_c}{\xi_{m-1,0} \cdot H_{m0} \cdot \gamma_b \cdot \gamma_f \cdot \gamma_\beta \cdot \gamma_v}\right) \quad (5)$$

with a maximum of: 
$$\frac{q}{\sqrt{g \cdot H_{m0}^3}} = 0.2 \cdot \exp\left(-2.6 \frac{R_c}{H_{m0} \cdot \gamma_f \cdot \gamma_\beta}\right) \quad (6)$$

where  $\alpha$  = slope angle;  $\xi_{m-1,0}$  = breaker parameter based on the spectral period  $T_{m-1,0}$ ;  $\xi_{m-1,0} = \tan \alpha / (2\pi H_{m0} / (g T_{m-1,0}^2))^{0.5}$ ;  $\gamma_x$  = influence factor, see EurOtop (2007) for more information. Equation 5 generally represents plunging or breaking waves on gentle slopes. In contrast, Equation 6 - the maximum overtopping - describes surging or non-breaking waves on fairly steep slopes. The reliability of Equations 5 and 6 is described by a standard deviation ( $\sigma$ ) in the exponent respectively  $\sigma(4.75) = 0.5$  and  $\sigma(2.6) = 0.35$ .

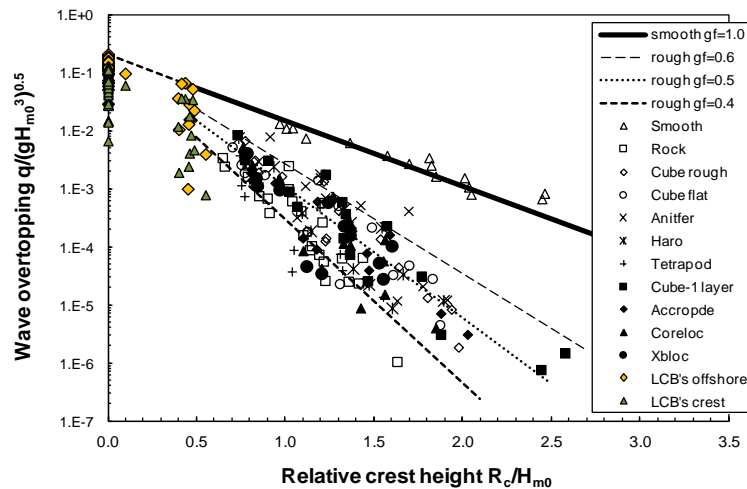


Figure 1. Wave overtopping discharges for rubble mound structures, CLASH and LCBs.

The overtopping for rubble mounds (CLASH, 2004) are given in EurOtop (2007), Figure 6.6, repeated here as Figure 1. In this graph Equation 6 (the bold solid line) describes very well the results of the smooth 1:1.5 structure, which was tested as a reference in the CLASH-work.

In Figure 1 the overtopping discharges are also given for the LCBs. The overtopping discharge for these structures has been calculated and not measured. Distinction is made between overtopping discharge at the crest and farther across the crest where some water has been absorbed by the permeable structures. The results of the LCBs show the average trend that should be expected, and the data on the crest indeed give a little smaller overtopping discharge. For zero freeboard it is expected that the overtopping discharge will be lower than the prediction by Equation 6 (represented by the dotted line between  $R_c/H_{m0} = 0 - 0.5$ ). This is indeed the case for the data points.

## Determining b-Values for Distributions of Overtopping Wave Volumes

Figure 2 shows three examples of distributions of overtopping wave volumes given on a Weibull scale, where  $V_{bar}$  is the average overtopping wave volume and  $P(V)$  is the probability of exceedance. The distribution is a Weibull distribution if the data show a straight line on a graph like in Figure 2. Very often this is not the case and the lower part shows another inclination than the upper part (see for instance the data for 1.32 s and for the LCB). As we are mostly interested in the largest overtopping volumes, the distributions were fitted on the extreme tail that includes the largest volumes. Data are included in the fitting so long as the data seem to follow a straight line fitting the upper part of the distribution. The inclination of the line gives the b-value (shape factor) of the distribution. See EurOtop (2007), Figure 4.3 for the effect of different values of the shape factor.

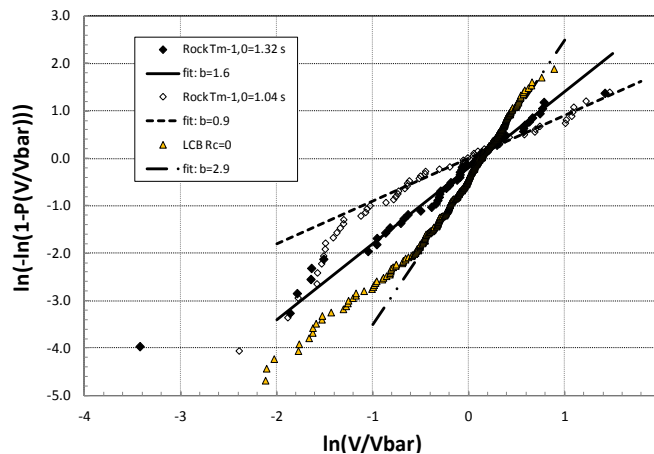


Figure 2. Fitting to the largest overtopping wave volumes on a Weibull-plot.

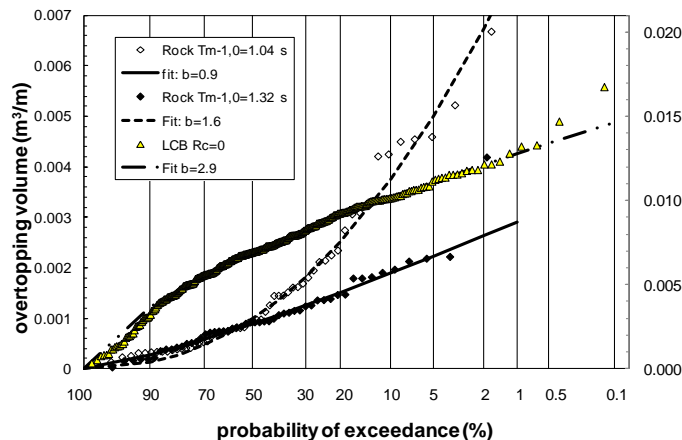


Figure 3. Data of Fig. 2 on Rayleigh scale (a straight line gives a Rayleigh-distribution).

The fitting on Figure 2 gave  $b = 1.6$  for a wave period of  $T_{m-1,0}$  of 1.32 s, which is quite close to a Rayleigh-distribution ( $b = 2$ ). For  $T_{m-1,0} = 1.04$  a much steeper curve was found with  $b = 0.9$ . These two examples belong to the CLASH data. The LCB with the crest at the water level gives  $b = 2.9$ , which is much flatter than a Rayleigh distribution. Figure 3 gives the three distributions on a Rayleigh scale and shows clearly the influence of the  $b$ -value on the distribution (LCB data on the second vertical axis at the right).

### Analysis of the $b$ -Values

Figure 4 shows the data for smooth slopes described in Hughes et al. (2012), resulting in a relationship between  $b$  and the relative freeboard  $R_c/H_{m0}$ :

$$b = [\exp(-0.6 R_c/H_{m0})]^{1.8} + 0.64 \tag{7}$$

The smooth 1:1.5 data of CLASH (see Figure 1) are also shown in Figure 4. The CLASH data fit fairly well with the EurOtop line with  $b = 0.75$ , although there is a slight tendency for just larger values.

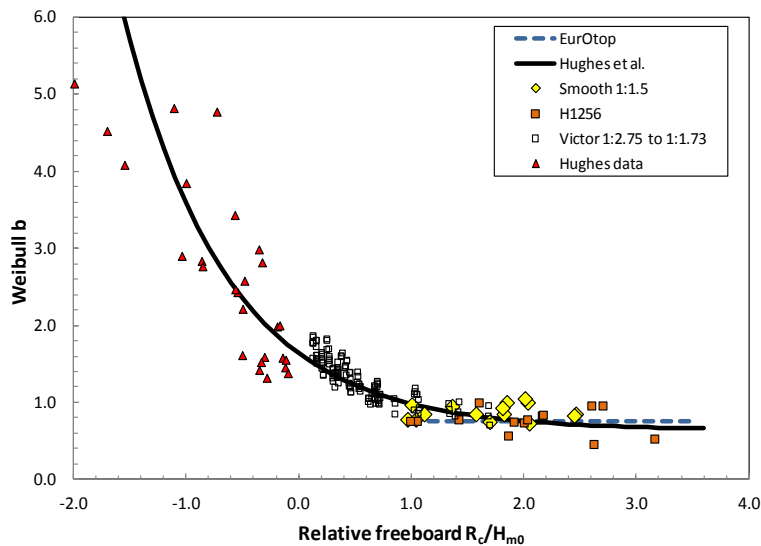


Figure 4. Data and relationship for smooth structures (Hughes et al., 2012) with the smooth structure data of CLASH

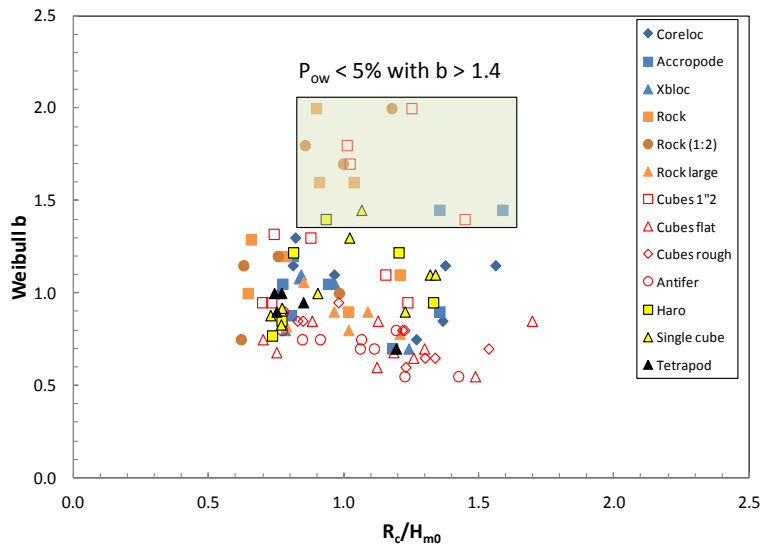


Figure 5. CLASH-data for rubble mound structures as a function of  $R_c/H_{m0}$ .

Figure 5 proposes a zoomed plot of Figure 4 including only the CLASH rubble mound structures. The majority of the data gives  $b$  in the range 0.5-1.4, but in some cases  $b$  exceeds 1.4. A closer examination indicated that these higher values correspond to wave conditions with less than 5% of overtopping waves, see Figure 6, and mainly associated to large wave steepness (short periods). This condition can also be seen in Figures 2 and 3, where the short period of 1.04 s (with 5% overtopping percentage) gives a  $b$ -value of 1.6. Overall there seems a slight tendency that single layer units have a somewhat larger  $b$ -values than armour units placed in two layers. But it is more a slight tendency than a consistent trend. Farther on in the paper the data will be compared with the full trend for smooth structures.

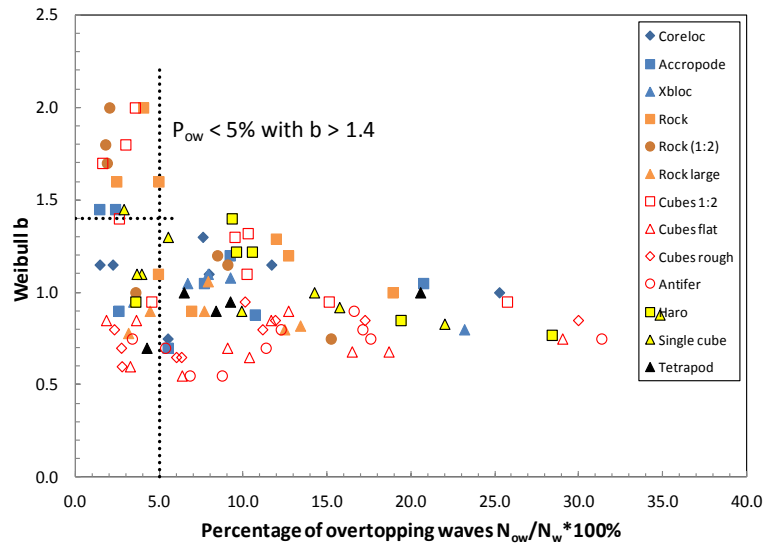


Figure 6. CLASH-data for rubble mound structures as a function of percentage of overtopping waves.

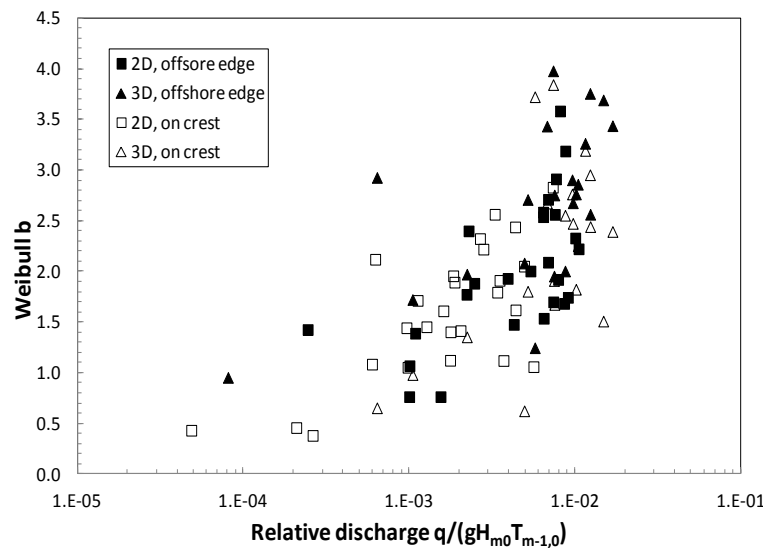


Figure 7. Data for LCBs.

The overtopping for LCBs was calculated from the records of two wave gauges on the crest. Between the gauges water will percolate into the rubble mound structure, which causes less overtopping for the wave gauge on the crest compared to the gauge at the seaward edge. But as a smaller overtopping discharge could give also a smaller  $b$ -value (based on Figure 4), the data of the two different gauges may well fall in the same range. This is indeed the case as shown in Figure 7. Similar symbols (open or closed) can be

compared and actually the whole data set shows a similar trend of increasing b-value with increasing overtopping discharge. The horizontal axis will be explained later on.

## Comparison of Rubble Mound Structures against Smooth Impermeable Structures

The first comparison of rubble mound with smooth structures will be on repeating Figure 4 with the CLASH and LCB data, see Figure 8. The LCB data exhibit substantial scatter as the majority of the data was measured at  $R_c = 0$ . The CLASH data for conventional rubble mound structures appear around the given trend, except for some of the data with less than 5% overtopping waves. The data cloud also does not reach the right end of the graph as for  $R_c/H_{m0}$ -values larger than 2 there will be no overtopping. Overall one may conclude that using  $R_c/H_{m0}$  might be sufficient for smooth structures to describe the b-value, but it is obviously less accurate for rubble mound structures.

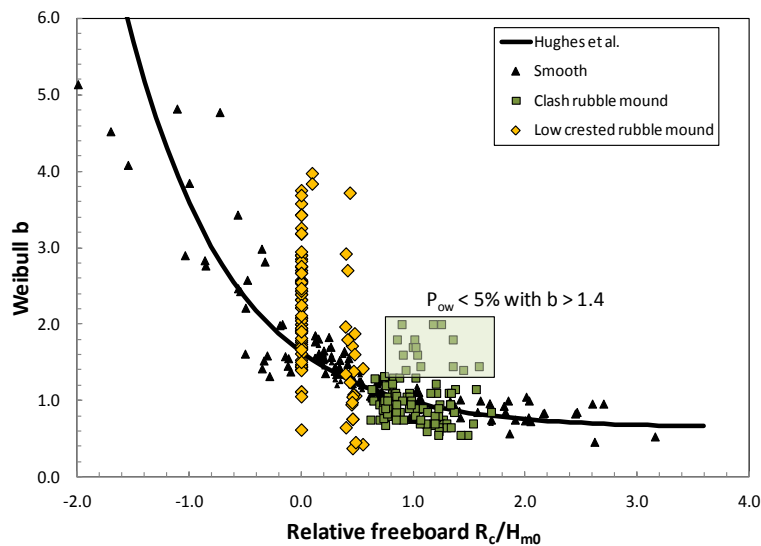


Figure 8. Comparison of rubble mound structures with smooth ones against  $R_c/H_{m0}$ .

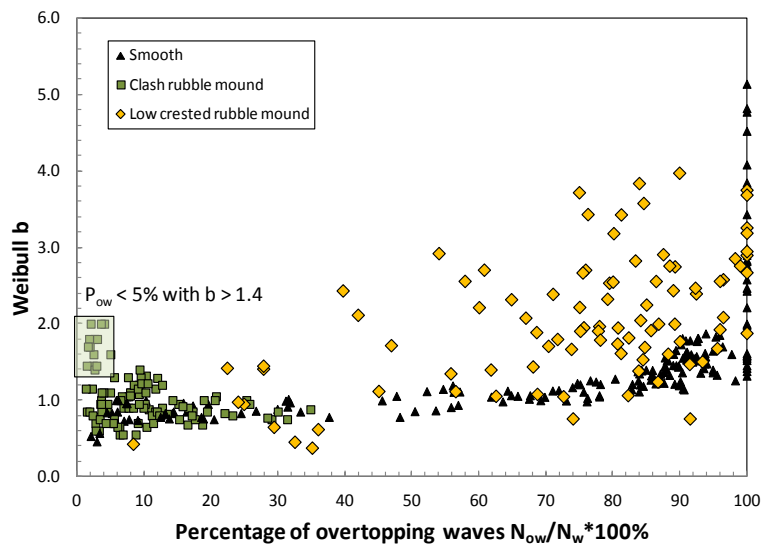


Figure 9. Comparison of rubble mound structures with smooth ones against percentage of overtopping waves.

If the trend is that b-values increase for increasing wave overtopping, then there are more dimensionless

parameters that can describe this effect. One of them is the percentage of overtopping waves  $N_{ow}/N_w * 100\%$ . In this case the b-value for 10% of overtopping waves appear at the same location, regardless if the data was measured at rubble mound or smooth structures (so differently from Figure 8). Figure 9 shows the graph. There is also clearly a drawback: all data for submerged or very low-crested structures will appear at 100%. Moreover, the LCB-data give a large scatter. One can conclude that the percentage of overtopping waves does not describe the whole trend sufficiently.

Another parameter which may give a continuous trend is the relative discharge, very often written as  $q/(gH_{m0}^3)^{0.5}$ . It is hypothesized that if the relative discharge is similar for two different structures, the wave volume distribution may also have the same shape factor b. Also, the relative discharge implicitly includes the combined effects of relative freeboard, wave steepness, and structure slope. Figure 10 presents the data. The CLASH-data cover the same range as the smooth structure data with relatively minor wave overtopping ( $q/(gH_{m0}^3)^{0.5} < 0.01$ ). These data give more scatter than seen for smooth structures, but fall around the smooth data with a slight tendency of giving larger b-values. As before, the exception is some of the data with less than 5% overtopping waves.

There is a sharp increase in b-value when the crest is close to the water level and this increase continues for submerged structures ( $q/(gH_{m0}^3)^{0.5} > 0.1$ ). Now the LCB-data show the same trend as the submerged structures of Hughes and Nadal (2009), at the right side of Figure 10. There is more scatter and again a slight tendency of giving larger b-values.

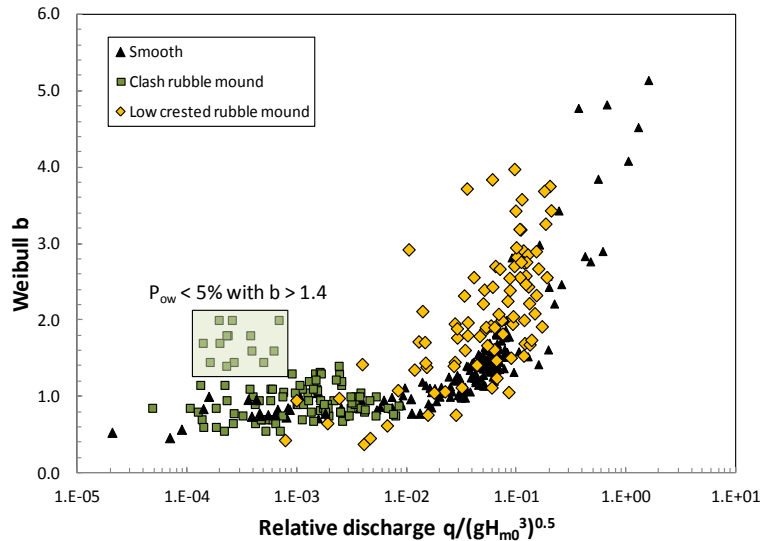


Figure 10. Comparison of rubble mound structures with smooth ones against relative discharge  $q/(gH_{m0}^3)^{0.5}$ .

Figures 2 and 3 showed the tendency that sometimes a smaller wave period gave larger b-values. Also the large b-values for the CLASH-data with less than 5% overtopping waves were often due to small wave periods. Another dimensionless parameter which still gives relative discharge is  $q/(gH_{m0}T_{m-1,0})$ . In Hughes and Nadal, 2009, this relative discharge is given, but then with the peak period  $T_p$ . Figure 11 plots all data against this relative discharge. Comparison with Figure 10 shows that this relative discharge may result in a slight reduction in scatter. In particular, smooth slopes show a nice trend with relatively small scatter, see Figure 12.

The new trend for smooth structures can be given by:

$$b = 0.73 + 55 \left( \frac{q}{gH_{m0}T_{m-1,0}} \right)^{0.8} \quad (8)$$

Equation 8 results in a conservative approach if applied to rubble mound structures as the rubble mound



data fall almost all above this curve. The following equation is derived from the data for rubble mound structures:

$$b = 0.85 + 1500 \left( \frac{q}{gH_{m0}T_{m-1,0}} \right)^{1.3} \quad (9)$$

by excluding the data falling in the shaded area of Figure 11 with  $P_{ow} < 5\%$  and  $b > 1.4$  as it is not yet understood why these data deviate so much from the average trend. Due to the large scatter of rubble mound data, the use of Equation 8 may however be preferred also for these structures.

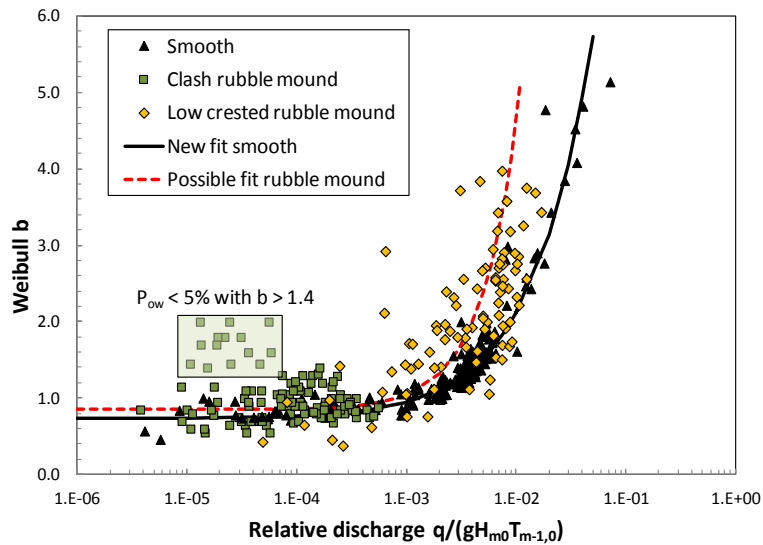


Figure 11. Comparison of rubble mound structures with smooth ones against relative discharge  $q/(gH_{m0}T_{m-1,0})$ .

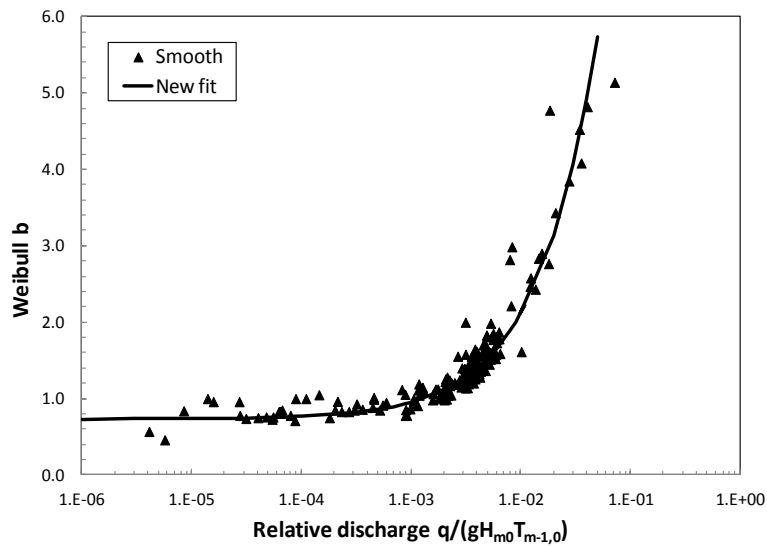


Figure 12. Smooth structures only against relative discharge  $q/(gH_{m0}T_{m-1,0})$ .

## Conclusions

The shape factor  $b$  in the Weibull distribution, describing overtopping wave volumes, has a large influence on prediction of maximum overtopping wave volumes. A small  $b$ -value means that the majority of the overtopping wave volumes will be small and that there will be a few very large overtopping volumes (steep tail on the curve). For a large value of the shape factor there will be more significant overtopping wave volumes with more or less the same volume.

Existing data on smooth and rubble mound structures show that there is a clear and rapid increase in  $b$ -value when the overtopping increases significantly for low-crested and submerged structures. The clear trend with the relative freeboard as dimensionless parameter is good for smooth structures, but it does not function well for rubble mound structures. The percentage of waves as a parameter describing the  $b$ -value does not work sufficiently for slightly emerged structures where all data will be given for 100% overtopping waves.

A continuous trend is found if the relative discharge  $q/(gH_{m0}^3)^{0.5}$  or  $q/(gH_{m0}T_{m-1,0})$  is used, and this trend may be due to the fact that relative discharge is a function of relative freeboard, structure slope and wave steepness. Now the  $b$ -value is directly coupled to the discharge and predicting formulae are given by Equations 8 and 9. Rubble mound structures give more scatter than smooth structures and there is also a small number of tests with low percentage of overtopping waves and large  $b$ -values which is not understood presently.

## Acknowledgments

With great pleasure we have received and used the data of the University of Ghent on very steep slopes and low freeboards (Peter Troch and Lander Victor).

The support of the European Commission through FP7.2009-1, Contract 244104 - THESEUS project ("Innovative technologies for safer European coasts in a changing climate"), [www.theseusproject.eu](http://www.theseusproject.eu), is gratefully acknowledged.

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